Discounting the Social Cost of Carbon

Master Thesis

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"Time is money" (anonymous)

"El tiempo es la sustancia de que estoy hecho. El tiempo es un río que me arrebata, pero yo soy el río; es un tigre que me destroza, pero yo soy el tigre; es un fuego que me consume, pero yo soy el fuego." (Jorge Luis Borges, 1946) [2]

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[2] "Time is the substance I am made of. Time is a river which sweeps me along, but I am the river; it is a tiger which destroys me, but I am the tiger; it is a fire which consumes me, but I am the fire."

1. Introduction

1.1 General introduction

Policy relevant studies of global changes that stretch over decades into the future can often only be framed in scenarios. In principle, all scenarios are testable, since time will tell. The first scenario studies appear in the last quarter of the twentieth century (Club of Rome, 1972) and are of course now open for a reality check. However, criticism on their simplified underlying assumptions and their limited mathematical formulations make these early studies obsolete already in their own time (Cole, Freeman, Jahoda, & Pavitt, 1973). Nevertheless, their worth is that they serve as a starting point in our thinking of the future state of humankind, which is nowadays more than an interesting but harmless philosophical, theological or entertaining topic of conversation. After four decades of intense debate, discussions about the reality of human-caused climate change have fallen silent: the climate is indeed changing by the human emission of greenhouse gases, in which CO_2 from fossil fuels forms the greater part (Cline, 1992). At present, the debate concerns mainly abatement strategies of the consequences of climate change. Because human use of energy is still based for 90% on CO_2-producing technologies, a huge effort awaits humankind in the coming decades, as put forward by the IPCC coming 6th assessment report.[3] How to implement those efforts? How to align opinions, needs, capabilities and opportunities? How to invest most efficiently climate change abatement? Most importantly, how do we weigh costs and benefits of today against those of the future? Recent jurisdiction of the German Federal Constitutional Court (Bundesverfassungsgericht, 2021) has lifted the scope of the last question from the moral into the juridical/political realm: "...eine zu kurzsichtige und damit einseitige Verteilung von Freiheit und Reduktionslasten zulasten der Zukunft verhindert werden."[4] It is evident than one needs the means to couple future costs and benefits to those of today.

In the economy, future costs and benefits –generally called values– can be expressed into today's values via the method of discounting. Different values of goods or services at different times can be compared to each other quantitatively, or at least prioritized, by translating the future values[5] into present values. Present value equals future value times a discount factor. In finance, one takes for the purpose of discounting usually the exponential function of the time

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[3] https://www.ipcc.ch/site/assets/uploads/2021/06/Fact_sheet_AR6.pdf

[4] "It is thus imperative to prevent an overly short-sighted and thus one-sided distribution of freedom and reduction burdens to the detriment of the future."

[5] In principle, the same holds for past values, but if they are recorded or can be linked to other past values, the recorded or linked values are usually taken.

difference multiplied by a discount rate. In business, the latter is equal to the cost of capital (Palepu, Healy, and Peek, 2019, p. 313). Also for public projects one uses discount rates, although they are often lower because they involve less risk than many private projects. Climate change abatement is a costly project for mankind, but it is not a financial question, even not an economic one. Nevertheless, some sort of discounting of future values is needed before one can decide on major abatement programs.

Financial markets operate with a broad range of time horizons. Nowadays, with ITC (information and communications technology) applied in finance throughout (*fintech* in short), the lower boundary could be milliseconds. The upper boundary is mostly set by the lifetime of major public infrastructure projects, which could surpass 100 years. Because of the different characters and uncertainties of the underlying subjects in finance (*e.g.*, personal consumption, houses, airports, commercial projects), interest rates are not time-span (or *maturity*) independent. Their term structure –*i.e.*, time dependence– is an economic variable by itself, quantified by the yield curve, which varies according to short, medium and long-term economic expectations.

The question in relation to climate change abatement is whether these financial and economic concepts suffice to lay the foundations of major investment decisions. Unfortunately, any small variation in the chosen discount function will swamp over long periods all other variations, literally muffling all discussion and stalling decision making. This extreme sensitivity on a rather technical issue is a major stumbling block in discussions of the cost of climate change. A second major stumbling block concerns the intractability of most climate and economic models, so called Integrated Assessment Models (IAMs). This thesis relates mainly to the former stumbling block: how to discount the future in view of climate change. Bypassing or resolving the intractability of many IAMs is an additional motivation of this work.

In this paper, I will analyze via an abstract but simple energy-climate-economy model the consequences of the use of various discounting methods on the present values of the costs of climate change. In particular, I will focus on the *externality* cost due to the damage to the global economy of present CO_2 emissions. This work builds on the publication by William Nordhaus (2017), in which he calculates a present (namely 2015) value social cost of carbon (SCC) of 31 \$[6] per ton emitted CO_2. For this purpose, Nordhaus applies the DICE Integrated Assessment Model (IAM) and current financial and economic data and knowledge of the atmosphere, including predictions until the year 2100. This result for the SCC deviates substantially from the estimate

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[6] 2010 international US dollar.

by Nicholas Stern (2006) of 198 \$[4] per ton CO_2. These two very different estimates impede proper debate and decision making.

The pivotal financial data in my study relate to discounting over long periods. The actual data of analysis are derived from a study by Giglio, Maggiori, and Stroebel (2015), who investigate the price differences of freehold (property rights are bought and sold for eternity) and leasehold (property rights are returned to the seller after a specified time) houses. The rapidity at which leasehold prices approach freehold prices for similar houses is an indicator of the general attitude of households towards the discount of, for them, expensive and durable goods over long periods.

In this study, I find that the discount function for the data of Giglio et al. (2015) can be well described by a power-law function[7]: $f(t) = (1 + t/t_0)^{-2}$, but also by a combined exponential–power-law function: $f(t) = \exp(-\beta t) . (1 + t/t_0)^{-1}$. Moreover, the term structure of interest rates recommended by the UK Treasury Board (HM Treasury, 2003) also fits both alternative discount function; the latter one with β = 0.58%/y and t_0 = 21 y. Moreover, I assert that these functional dependences have foundations in behavioral economics, in which hyperbolic discounting explains financial choices by individuals better than exponential discounting.

Furthermore, I investigate on an abstract level the sensitivities of the SCC on a few basic economic-financial parameters. If the use of carbon does not grow ($C \leq 0$%/y), this function is rather simple. It depends on the initial size of the economy (GDP) G_0 and its growth rate g, the natural CO_2 growth or decay rate h, the rate of pure time preference ρ, the inequality parameter η, and the damage parameter φ_2. Roughly: $SCC(g, h, C, \rho, \eta) = \frac{\mathrm{A}\, \varphi_2 G_0}{\rho + (\eta - 1) g - h}$, where A is a proportionality constant.

The power-law discount function can well describe the freehold-leasehold data of Giglio et al. (2015) and can explain some differences in estimates of the SCC by Stern (2006) and Nordhaus (2017). Nevertheless, its application is mathematically perilous because infinities can arise in case of prolonged positive exponential economic growth. I conclude that a large part of the controversy between Stern and Nordhaus can be explained by different choices for these parameters, stemming from fundamentally different but not mutually exclusive viewpoints.

The SCC depends strongly on rate of pure time preference ρ. The latter is a parameter subject to public debate, which cannot be settled by economics, but rather by social activism and jurisdiction. Recent example is the order by the German Federal Constitutional Court

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[7] Here, t stands for time.

(Bundesverfassungsgericht, 2021). Although the SCC covers the externality of global economic damage, it does not cover non-quantifiable or non-quantified damages to mankind, its culture and the earth's biosphere, nor does it include the (economic) effects of sudden transitions or societal disruption when tipping point temperatures are reached.

1.2 Significance of the study

The costs of climate change abatement are often compared to the social cost of carbon, SCC. Current estimates of abatement costs amount to trillions of dollars in the coming decade. An example of an important measure is the European Trading Scheme[8] with tradable allowances for CO_2 emissions by major carbon-intensive industries in Europe. However, when a benchmark is in use that is based on uncertain principles or data, then many current and future investments might become unpredictable, their rentability being uncertain, even prone to societal disaccord. This paper aims to unravel the underlying relationships of the SCC and to provide an alternative method of discounting. In this way it counteracts stiffening of the debate caused by ill-understood or unspoken differences in moral, psychological or rational principles.

The paper is organized as follows. Chapter 1 sketches the motivation of the paper and presents the main results of the study. Chapter 2 depicts the background needed to understand the cause and effects of climate change plus a literature review with a focus on the debate on discounting the costs of climate change, initiated by the Stern report (Stern, 2006). It concludes with the formulation of the research questions. Then, the next chapter (3) presents the theory and methods used. The results of various basic calculations are given and interpreted in chapter 4, whereas the last chapter (5) discusses the findings and concludes the paper.

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[8] https://ec.europa.eu/clima/policies/ets/markets_en

2. Background, Literature Review and Research Question

2.1 Human energy use and the climate

The consumption of energy is strictly speaking a misleading term. When one consumes an apple, the apple ceases to exist. Parts of its constituting material is taken up by the body of the consumer. Energy itself does not appear nor disappear, but transforms from one type to another. For instance, we transform the chemical energy in the molecules of natural gas into heat, making our houses more comfortable. However, when the heat has leaked out of the house, we regard it as lost, non-existing. After the final step, the original energy still exists, often as heat in the environment. In economic models, one distinguishes usually only primary and secondary/final energy, see Fig. 1.

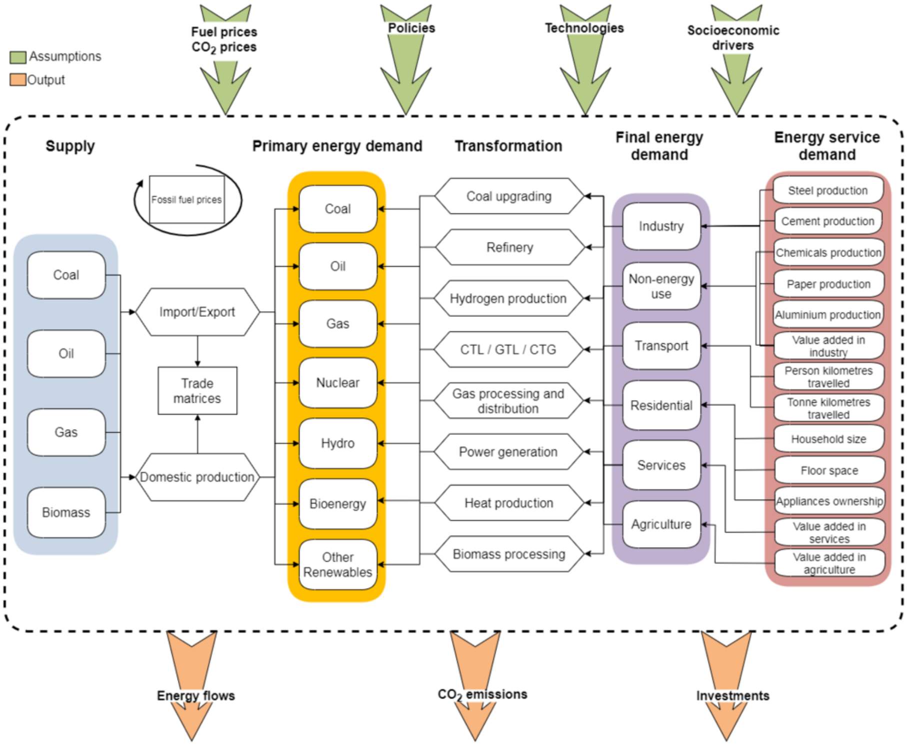

Figure 1. The embedding of primary and secondary uses of energy (IEA, 2020).

Figure 2 shows the sources of human primary energy after 1850. In addition, the growth of the population is shown (red dashed curve, axis at right). Because a high fraction of the sources is based on the burning of carbon-containing fuels, large quantities of CO_2 are produced. In almost all cases, this waste product is emitted into the atmosphere. The measured or estimated CO_2 concentration of the atmosphere since 1750 is depicted in Figure 3. From the pre-industrial level of 278 ppm[9] it rises by about 50 ppm until 1950. In the next 70 years another 100 ppm is added.

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[9] ppm = parts-per-million; 100 ppm = 0.01 %.

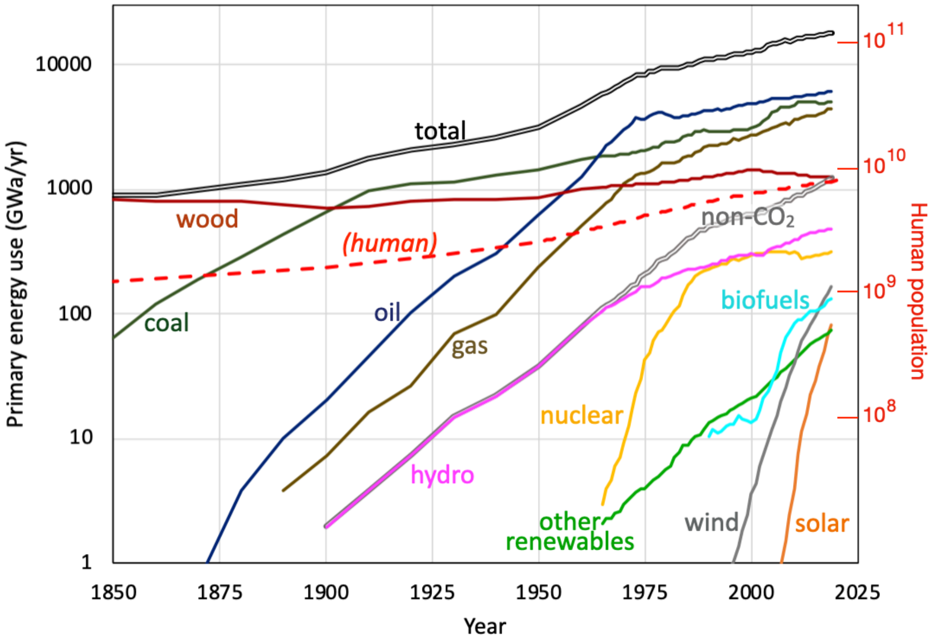

Figure 2. Primary energy sources 1850-2020.[10] All sources introduced after 1900 do not emit CO_2 gas; the 'non-CO_2' curve shows their sum. The red dashed curve is the estimated human biological energy production (not included in total), calculated with 150 Watt per capita (Davidovits, 2008, p. 146) the human population from 1.2 billion in 1850 to 7.8 billion in 2020 (see scale at right).

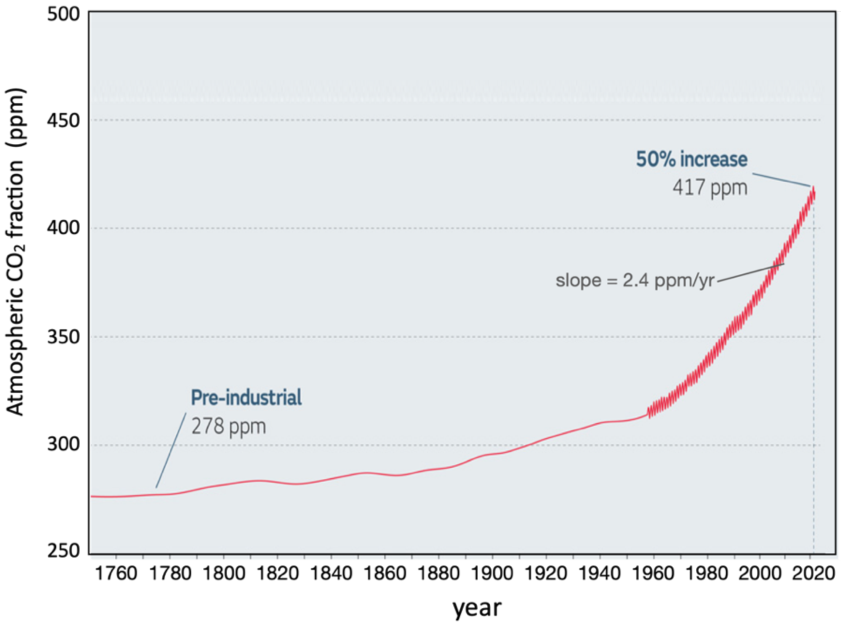

Figure 3. Atmospheric CO_2 fraction since 1750.[11]

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[10] From: https://ourworldindata.org/grapher/global-primary-energy?country=~OWID_WRL

[11] https://www.carbonbrief.org/met-office-atmospheric-co2-now-hitting-50-higher-than-pre-industrial-levels

Extrapolating figures 2–5 to the end of the 21st century does not look reassuring. However, neglecting all details, only three of these curves are in fact important: the total primary energy use, the human population, and the curve of non-CO_2 energy sources. These are, together with the evolution of the global gross domestic product (GDP) and the growth of energy conversion technologies, the major determinants of CO_2 emission. Note also that not only energy-related CO_2 emission determines the human-caused greenhouse effect[12]. CO_2 is also released in the production of cement and by deforestation. Another strong greenhouse gas is methane (CH_4), that originates from farming[13], decaying organic soils or thawing tundras. The contribution of

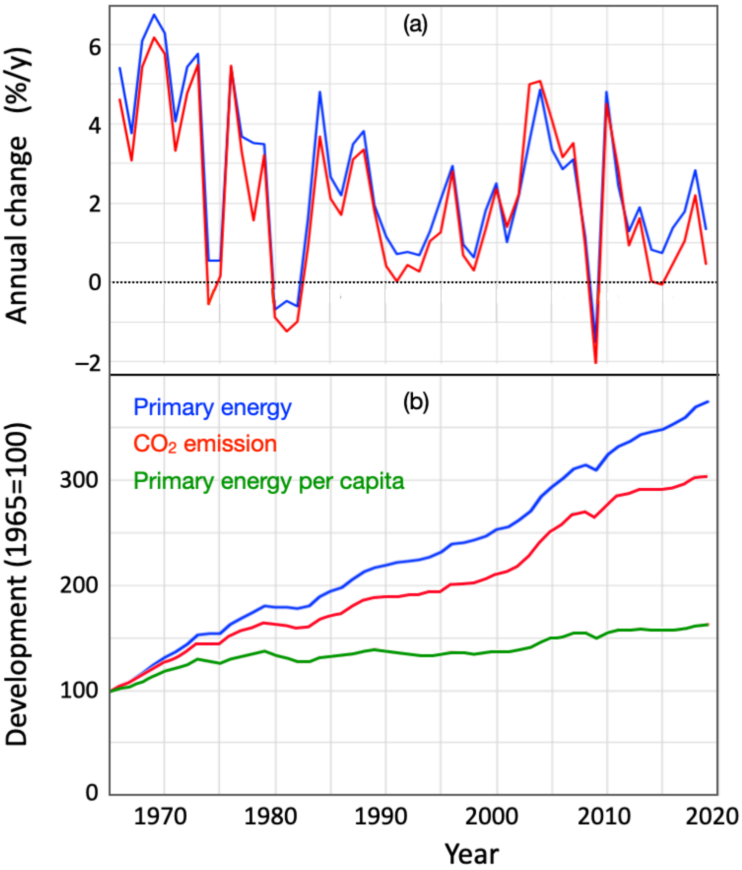

Figure 4. Annual change in global primary energy use, CO_2 emission, and primary energy use per capita, 1965-2020. (a): annual change, (b): development since 1965 (British Petroleum Company, 2020).

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[12] The natural greenhouse effect is about 33 °C (Cline, 1992), mainly due to water vapor and CO_2.

[13] Of course, food contains energy. Generally, all biological energy is not taken into account in climate change considerations because its primary source is the sun. On the other hand, transport of farm products, energy use for the machineries in farming and for the production of fertilizers, are included.

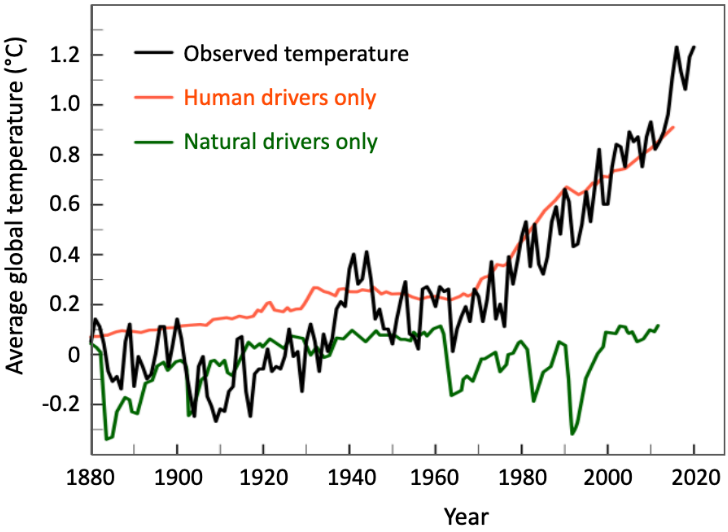

Figure 5. Average global temperature, 1880-2020 (modified from Knutson, Kossin, Mears, Perlwitz, & Wehner, 2017).

methane and other greenhouse gases are expressed in equivalent gigatons (Gt) of CO_2 (*i.e.* CO_2e). Of course, there are natural sinks of CO_2 as well: *e.g.* the oceans and vegetation. Finally, it is worthwhile to mention that the average primary human energy use is now 2400 W/capita, though there are large global variations, from 700 W/capita in India to 9900 W/capita in North America. In comparison, the human's physiological energy use is about 150 W (Davidovits, 2008, p. 146).

The costs and usefulness of the present and future energy system depend obviously strongly on the technologies of energy harvesting, transformation and distribution. Naturally, the technologies to exploit these sources change over time, see Fig. 1. Locally, sources become depleted, more efficient or less noxious alternatives emerge, sometimes heralded incessantly – such as nuclear fusion, sometimes developed relatively fast –such as nuclear fission.

The present value of all expected costs of the global energy supply system in this century depends strongly on the expected growth rate of the GDP, on the carbon use in the economy, and on the discount of future costs and revenues. The GDP growth rate and the decarbonization of the economy depend on the development of new and existing technologies, but might become affected by a changing climate. Note that carbon use of the economy is expressed as the amount of CO_2 emission of per unit of GDP. Decarbonization refers to the reduction of this ratio. Both the GDP growth and the decarbonization rate are difficult to predict over decades, moreover they are interdependent.

Despite these large uncertainties in developments and expectations, awareness of long-term consequences of actions or lack of actions now is of utmost importance for long-term conditions of life on earth (Rockström et al., 2021).

2.2 Greenhouse gas emissions

The fraction of atmospheric CO_2 rises steadily since the industrial revolution, see Fig. 3. The growth of the CO_2 concentration is nowadays 2.4 ppm per year. One can see in Fig. 4 that the growth rate of CO_2 emission has dropped to about 1% per year. In absolute terms it is now about 40 Gt/y in 2020, see Fig. 6, including the contribution from other greenhouse gases. In this figure three possible pathways are depicted after 2018 for a remaining budget of 600 Gt CO_2e (full curves) and one for a budget of 800 CO_2e. Climate models predict that with these limits, the temperature change until the end of the century can be limited to 1.5 – 2 °C. The lower budget with an annual emission of 40 Gt leaves 15 years to reduce almost all emissions.

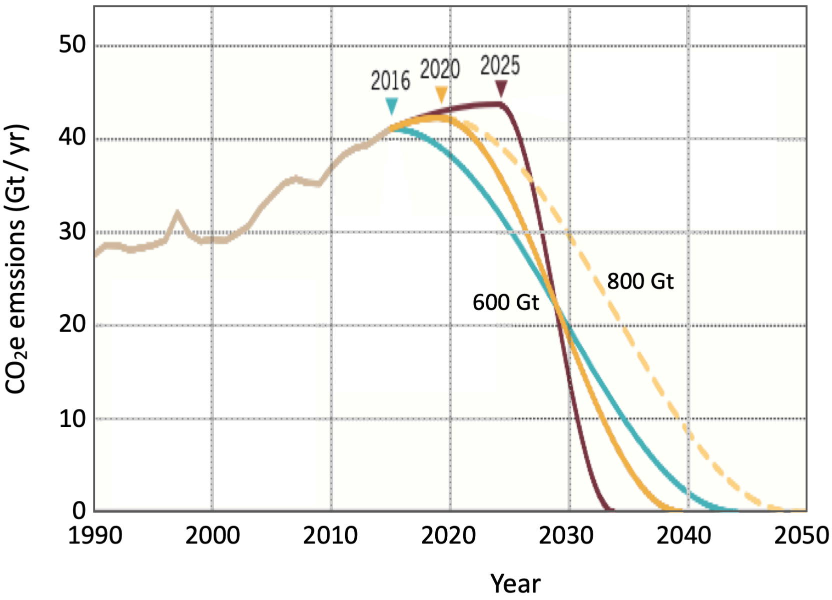

Figure 6. Annual greenhouse gas emissions, expressed in effective CO_2e, with four suggested scenarios for a remaining budget of 600 or 800 Gt, that could limit total global temperature increase to 1.5 – 2 °C. (1 Gt = 10^{12} kg). (Adapted from Figueres et al., 2017.)

2.3 The ethical question

In their treatise of the social cost of environmental damage and economic cost of abatement, Ferrari & Mery (2008) distinguish two types of future discounting. They base their ideas on the *Imperative of Responsibility* of Hans Jonas (1979): *Handle so, dass die Wirkungen deiner*

Handlung verträglich sind mit der Permanenz echten menschlichen Lebens auf Erden[14]. In particular, Ferrari & Mery (2008) write *Au sein du temps économique, s'il existe des préférences inter-temporelles, l'actualisation se justifie parce que les moments du temps ne sont pas considérés comme équivalents (perspective utilitariste). Au sein du temps irréversible (i.e. temps historique), des contraintes écologiques et éthiques peuvent être posées préalablement à toute décision économique.*[15] Thus, they place economic arguments irrefutably on a subordinate level in major and complex environmental problems, even though their costs aspects are rarely irrelevant. Therefore, discount rates in long-term environmental costs calculations have to be determined by sustainability principles, in particular by the principle of intergenerational equivalence.

2.4 The discount discussion in the climate-change debate

In 2006, the government of the United Kingdom commissioned an independent committee to gather and evaluate the current knowledge of the economics of climate change in the new century. The committee's 700-page report becomes known as the Stern Review (Stern, 2006). It states explicitly that climate change is the greatest and widest-ranging market failure ever. The authors argue strongly for a policy on climate change, that is based both on a disaggregated approach to confront consequences and on broad ethical standards. The Review meets fierce criticism and the first author, Stern, publishes a follow up paper (Stern, 2008) in which he addresses the alleged shortcomings of the Review. According to him, one must regard mitigation as an investment, a cost incurred now and in the coming few decades to avoid the risks of very severe future consequences. Despite the detrimental long-run effects, the review committee notices that emission of CO_2 and other greenhouse gases is accelerating (see *e.g.*, Fig. 4), mainly because fast-growing economies invest heavily in high-carbon infrastructure and because global demand for energy and transport increases. As a consequence, an atmospheric level of 550 ppm CO_2e [16] could be reached as early as 2035. Integrated atmospheric models show a high probability (69%) for the global average temperature to exceed 3 °C at this level. Continuous use of fossil energy sources in 21st century could eventually raise the temperature by 5 °C. Integrated

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[14] "Act in such way, that the effects of your actions are compatible with the permanence of genuine human life on Earth."

[15] "In the scope of economic time, if there exist intertemporal preferences, discounting is justified, because the various moments of time are not being considered as equivalent ('utilitarian perspective'). In the scope of the irreversible time (*i.e.* historic time), ecological and ethical constraints can be placed before of every economic decision."

[16] CO_2e : CO_2 equivalence. The emissions of other greenhouse gases are converted into equivalent CO_2 emission, according to their contribution to the greenhouse effect, relative to that of CO_2.

assessment models (IAM), the committee writes, provide tools for estimating the total impact on the economy. The Review and the follow up paper make estimates of the economic effects using the PAGE2002 IAM, which not only takes average expected temperature rise, but also risks of abrupt large climate changes into account. The results show that the impact is likely higher than previously thought. With 5 – 6 °C warming, the loss in global GDP amounts to 5 – 10%, with the less developed countries suffering losses even above 10%. The Review expresses the opinion that policy on climate change must be largely about reducing these risks. Moreover, IAMs must include also ethical judgements on the distribution of income and on the treatment of future generations.

The concept of discount rate relates both to pure financial factors –*e.g.* return on capital, real interest rate, opportunity cost of capital– and to the relative weight households attribute to economic welfare –or utility– in the future, similar to Ferrari & Mery's (2008) distinction. The latter meaning –in this paper called the *rate of pure time preference* (Gollier, 2013)– can also be applied to future generations. A rate of pure time preference of 0 means that all future generations are treated equally as present generations; a positive time discount rate means that the welfare of future generations is regarded as less important; their welfare is discounted in the eyes of the present generations. This issue, however, is not only economic, but also, if not mainly, ethical, as Jonas (1979) and Ferrari & Mery's (2008) express. The later Nobel Prize winner Maurice Allais writes in the appendix of his book *Économie et intérêt* (Allais, 1947) "...that balancing the interest of different generations is an ethical or political problem, in which the competitive market solution has no valid claim to moral superiority over other solutions that depend on action by the state", cited by Koopmans (1965, p. 2).[17]

Also Roy Harrod (1948) and Frank Ramsey (1928) regard inequal weights for the welfare of present and future generations as ethically indefensible. On the other hand, Péter Bauer (1957, pp. 112-126) puts emphasis to private savings decisions of the present generation and rejects governmental regulations or taxes to balance present and future interests. Despite his preference for the former standpoint, Tjalling Koopmans (1965) poses that economic considerations must come first in order to quantify the interests of different generations, otherwise a balanced consideration cannot be made. This technical notion is the economist's role in quantifying transgenerational solidarity. He introduces the over-time integrated utility function (u), for which an optimal time path must be found. If there is a guaranteed minimum

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[17] Also Paul Samuelson wrote about overlapping generations. "An exact consumption-loan model of interest with or without the social contrivance of money" J. Polit. Econ. Dec. 1958, 66, pp. 467-82.

of technical progress, the ethical principle of neutrality between generations leads to a positive time discount rate. Moreover, Koopmans states (1965, p. 34) that *it (discount rate) would have to be revised upward if it is estimated that technological progress will accelerate to such an extent to "overtake it", and could be revised downward if it is expected that progress will slow down*. In simpler terms, the applied discount rate s should include the economy's growth (or decline) rate, in line with Ramsey's (1928) equation:

$$s = \rho + \eta . g \quad , \qquad \text{(Eq. 1a)}$$

where s is the discount rate, ρ is the rate of pure time preference, η is a factor that captures aversion to inequality, and g is the growth rate of the economy.

Both Stern's publications discuss the appropriate application of discount rates to calculating the present values of future investment and damages. Though market discount rates abound, discounting the world of our descendants is, Stern says (2006, 2008), an ethical issue. He opts for a numerical value for s of 1.4% discount rate per year, which covers a rate of pure time preference ρ of 0.1% plus 1.3% of expected GDP growth g. A rate of pure time preference of 0% would put all the interest of all coming generations on an equal level, thus following Jonas' *Imperative of Responsibility* (1979) and Ferrari & Mery's (2008) distinction between economic and historic time. However, argues Stern, there is a change of say 10% that humanity will succumb in 100 years, lifting the rate of pure time preference to 0.1%.

After considering a number of additional uncertainties (*e.g.* climate feedback mechanisms, heavy burden on less developed regions), the Stern Review concludes that a business-as-usual policy would increase the total cost climate change to a permanent reduction in consumption per capita of about 20%. On the basis of their PAGE AIM model and that of others, the Review concludes that stabilization of greenhouse gases at levels of 500 – 550 ppm CO_2e to keep temperature rise below 2 °C will cost on average around 1% of annual global GDP by 2050. For that purpose, three elements of mitigation are essential: a carbon price, technology policy, and the removal of barriers to behavioral change.

Several economists swiftly raise criticism against the Stern Review, in particular Nordhaus (2007) and Dasgupta (2007). The three most profound concerns include 1) the discount rate; 2) the treatment of risk and uncertainty; and 3) the comparison between costs and benefits. While acknowledging some weaknesses, Ackerman et al. (Ackerman, 2007; Ackerman, Stanton, Hope, and Alberth, 2009a; Ackerman, DeCanio, Howarth, and Sheeran, 2009b; Ackerman, Stanton, and Bueno, 2010) defend the report, lauding the Review's link between economic analysis and the urgency of the climate change problem. Nordhaus (2007) criticizes Stern's discount rate of 1.4%, a value inconsistent with current market real interest rates and savings rates. Nordhaus writes that such an extremely low discount rate magnifies unjustly impacts in the distant future and rationalizes deep cuts in emissions, and indeed in all consumption, today. Nordhaus bases his argument on findings from *a.o.* scenario calculations with his own DICE model.[18] These findings show that optimal economic policies to abate climate change involve modest emission reductions in the short run, followed by large reductions in the medium and long runs.

In his critique on the Stern Review, Nordhaus does not ignore the ethical or philosophical argument for a finite discount rate, but he distances himself from Koopmans and Ramsey's original view. He writes (2007, p. 692) *This approach does not make a case for the social desirability of the distribution of incomes over space or time of existing conditions, any more than a marine biologist makes a moral judgment on the equity of the eating habits of marine organisms in attempting to understand the effect of acidification on marine life*. In other words, the economists have not only the prior role (of assisting decision makers by providing the needed background knowledge), but also a moral one: the description of the market economy becomes a prescription for decision making, bypassing Jonas' imperative, a distinction that Partha Dasgupta stresses (2007). Furthermore, Nordhaus questions the utility function, which could be indefinable over long time spans. The utility function u could be expressed as the consumption c to some power η (the latter is the elasticity of marginal utility of consumption, sometimes called the equality parameter). Also the choice for $\eta = 1$ in the Stern Review is debatable and affects present costs of future climate change dramatically.

Finally, Nordhaus denounces (2007, p. 701) *Rather, it depends decisively on the assumption of a near-zero time discount rate combined with a specific utility function. The Review's unambiguous conclusions about the need for extreme immediate action will not survive the substitution of assumptions that are more consistent with today's marketplace real interest rates and savings rates. Hence, the central questions about global-warming policy—how much, how fast, and how costly—remain open.*

Also Weitzman (2007), Dasgupta (2007) and other economists criticize either mildly or sternly the Stern Review because of its unrealistic low discount rates or too simple utility functions. According to Martin Weitzman (2007), the review points out justly that interest rates for discounting climate change are pivotal but uncertain. Present market values of 6 or 7%/y are in his opinion irrelevant and more appropriate seems rates between 2 and 4%/y. But more importantly, he notes that spending money to reduce global warming should not be aimed at smoothing consumption, but at buying insurance to offset the small change of a ruinous

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[18] Documentation of the DICE model are available via https://williamnordhaus.com/dicerice-models

catastrophe. He proposes to include in the Ramsey formula for the discount rate also the uncertainty of the climate change and of its effects on economy and society in general. He proposes the addition of a variance term σ^2 in the Ramsey formula:

$$s = \rho + \eta . g + \left(1 - \frac{1}{2}\eta\right) . \eta . \sigma^2 \quad . \qquad \text{Eq. (1b)}$$

Others give extensions of the Ramsey equation along the same lines (*e.g.* Gollier, Koundouri, and Pantelidis, 2008; Ackerman et al., 2009b). The stochastic modification by Gollier et al. (2008) incorporates uncertainties in the socially efficient discount rate that are related to possible shocks (or 'regime switching') of the growth rate of consumption or of short-term interest rates. In particular, he calculates the social cost of carbon using declining discount rates (DDRs) from 4.5%/y initially to 1.5%/y after 400 years. He applies this concept to the FUND IAM (Tol, 1997) and Stern's PAGE model plus the baseline scenario of the Stern Review, to calculate present-values of *a.o.* the benefits of CO_2 emissions reduction, up to 400 years ahead. He finds roughly a ten times lower SCC than Stern with a constant 1.4%/y discount rate.

Dasgupta (2007) and Buchholz & Schumacher (2010) conclude that the authors of the Review advocate a strong and immediate action on climate change, not because of new climatic facts, but as a result of their views on intergenerational equity. However, they question the legitimacy of a purely intergenerational instead of a combination of inter- and intragenerational equity. They stress the need of also taking into account the well-beings of the poor in the contemporary world. In a formal way, their objection concerns the choice in the Stern Review to use a value for η of 1. Higher values than unity would promote a more globally equal distribution of welfare. As a consequence, the discount rate *s* will be higher, even with a low rate of pure time preference ρ close to zero. Dasgupta (2007, p. 6) remarks that *one should be very circumspect before accepting numerical values for parameters for which we have little a priori feel. One can’t get an intuitive feel for them from huge computer runs because it is usually not possible to track what’s influencing what in a sharp way.*

Similar considerations about the function of discount rates are already expressed much earlier by William Cline in a discussion essay (Cline, 1993), directed at the World Bank. In fact, Cline criticizes the World Bank policy that all environmental investment projects should be discounted at 8%, not exceptionally high for pure profit searching investors. He notes that with that 'high' discount rate, a fictitious extra consumption of \$1 (1992 price level) by his bicentennial predecessor would have impeded \$4.8 million worth of consumption by the current Chairman of the World Bank's Council of Economic Advisers. Cline (1993) proposes instead a 2% discount rate for climate change abatement projects, partly financed by lower consumption in developed societies and partly by capital. He rejects the World Bank standpoint (World Bank, 1992, p. 161) that only a minimal action on global warming is justifiable.

In contrast, many other economists dismiss the Nordhaus' moral standpoint. Examples are Ackerman (2007), Buchholz & Schumacher (2010), and Michl (2010).

In his defense to the Stern Review, Frank Ackerman (2007) not only comments the objections raised by the critics, but he also brings the original ethical notion of Ramsey and Koopmans to center stage. He writes explicitly (Ackerman, 2007, p. 6) *If climate change or other problems will make future generations worse off, the argument reverses itself: the present generation should do much more for its poorer descendants. If the lives of future generations will be sufficiently worse than ours, the discount rate could even become negative.* Frank Ackerman underwrites Weitzman's suggestion (2007) that the worst-case damages of greatest concern are inherently incalculable, but essential for current decision making. One might begin by asking, not what is the most likely value of the damages if the targets are achieved, but rather what is the worst case that is credible under those targets, *e.g.* the 98th percentile outcome? To elucidate his point, he refers to the high losses suffered in the New Orleans area during the hurricane Katrina in 2005, which casts doubt on the analytical framework of comparison of costs and benefits. Finally, Ackerman (2007) recasts the cost-benefit discussion into two fundamental questions: 1) What is the maximum atmospheric concentration of CO_2 at which unacceptable climate outcomes can be ruled out with a high degree of confidence? 2) What is the least-cost strategy for stabilizing at that concentration? The second question involves cost-effectiveness analysis: given the target, what is the cheapest way to get there? This approach is much more tractable than cost-benefit analysis, since it includes only costs, not benefits (or avoided damages) of mitigation.

From a neo-Marxist viewpoint, Thomas Michl (2010) criticizes Nordhaus' over-simplified proscriptive attitude grounded in neoclassical economics. He writes (Michl, 2010, p. 543) *Nordhaus is asking us to endorse the outcomes observed in real economies. The social planner should adopt the preference parameters extracted from market outcomes. (...) Nordhaus's position can now be seen as an evasion of some very difficult political questions that simply adopts the preferences of the financial and economic elites and assigns them to the social planner. Because the capitalist agents discount their own future generations (...), so should policy makers who presumably are charged with protecting the future of humanity.* Despite his critique on Nordhaus, also Michl poses the question whether the social planner would want to use normal saving behavior, with η equal to 1, to determine how much utility each generation should experience. In sum, Michl finds most perilous the fact that Nordhaus and others advise policy makers on neoclassical grounds to postpone investments in climate change abatement.

Also Koopmans (1960) and Arrow (1999) dismiss the utilitarian position, that Nordhaus (2007) accepts, on completely different grounds. If one does not discount utility far into the endless future, no time preference ordering[19] could satisfy the (weak) Pareto condition (Chichilnisky, Hammond, and Stern, 2020). Moreover, earlier generations should then make unreasonably excessive sacrifices. This deadlock can only be avoided by first including an exogenous background extinction process ensuring that with unity probability all life will cease in a finite time. This way, the Stern Review chooses arbitrarily a time preference rate of 0.1% per year. Chichilnisky at al. (2020) formalize this concept by stipulating that the discount remains bounded if and only if the expected discounted population is bounded by some finite number.

Eric Neumayer (2007) has a completely different standpoint, criticizing the principle of utility itself, elaborated by Koopmans (1965) and used in IAMS by Stern, Nordhaus and many others. Neumayer claims that we cannot know the preferences of future generations. Moreover, there is *no way of adequately valuing the utility loss from, say, the loss of glaciers, wetlands, forests and coral reefs, the damages to coastal, marine, arctic, mountain and other ecosystems and the likely massive rise in the rate of species extinction, which are all likely to be associated with temperature increases* (Neumayer, 2007, p. 299). Rather, one should respect the fundamental and inalienable rights of future generations. He argues that climate change above a minimal level violates these rights. He maintains that, despite the increased utility for future generations by continuous global GDP growth, there will be an irreversible, non-substitutable loss of natural capital. He concludes that this loss cannot be adequately monetarily valued. He considers cost-benefit studies and debates on discount rates out of touch with reality. Instead, ethical questions will heavily influence discussions and decision-making regarding climate change. Of course, efforts to add value to natural assets and their returns, thus to internalize externalities, should not be dismissed, see for instance the discussions in (Barbier, 2019) and (Cai, Judd, Lenton, Lontzek, and Narita, 2015) and the reviews by Atkinson, Bateman, and Mourato (2012) and by Bartosz, Lienhoop, and Hansjürgens (2015).

A similar critical view on the use of discounting in the climate change debate is expressed by Mark Buchanan, who writes (2011, p. 516) *What are currently considered the best analyses of some of the world's most pressing problems hinge almost entirely on quite arbitrary techniques for discounting the future*.

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[19] A time preference ordering is a function $f(t)$, that indicates the relative weight of the utility at time t to the present utility. Most known are the $\beta^{-1}\exp(-\beta t)$ continuous discounting preference ordering and the hyperbolic discounting $\left(1+\frac{t}{t_0}\right)^{-1}$.

After five years of intense scientific or non-scientific debate, the discussions on the methods and conclusions of the Stern Review continue. Nowadays the data in the Review are outdated, but its central message –repeated and further explained by Stern (2008)– is still part of the on-going debate. The increased recent interest in the debate might be related to the Paris 2015 accords, which –though not binding– start to serve as a reference point. A decade after the Stern-Nordhaus debate, the issue of neutrality of utility over generations is becoming the main theme in current thinking about climate change and future society, ecosystems or even life on earth. The ethical–economic controversy on future discounting cannot be resolved easily and could hinder debates on major environmental issues for decades. However, one might expect a Kuhn-like paradigm shift in our thinking (Kuhn, 1962) when novel empirical evidence appears, when still indeterminate ethical notions or dispositions toward our own and our descendant's state in the far future evolve (Krznaric, 2020; Bundesverfassungsgericht, 2021) or when –God forbid– major environmental disasters force us to make abrupt far-reaching emergency decisions. On the other hand, Chichilnisky et al. (2020) note that one should discuss policies that can manage change rather than discussing what “sacrifices” might be worthwhile. The role of scientists in the debate should, thus, not be to make *normative* statements about the best or most realistic options or technologies (Waisman, De Coninck, and Rogelj, 2019). Instead, they should provide insights and information to governments, private sector and society, strengthening a framework of possible mitigation and adaptation policies. Nevertheless, in my opinion science must also clarify where and how norms –being explicit or implicit– rather than insights and information affect policy relevant decisions. Hence, apart from providing more and better data, science should also investigate data selection and interpretation take place.

After the Paris accords, the world has set its goal to limit global warming to well below 2 °C, preferably below 1.5 °C. Although not a binding political agreement, this accord has at least focused the debate about mitigation and adaptation, one could say, to Ackerman's (2007) two fundamental questions, namely on the maximum acceptable atmospheric concentration of CO_2 and on the least-cost strategy to keep it at that level. Atmospheric chemists and physicists, climatologists, and many other natural scientists still debate the first one, though nowadays their discussions are mainly about monitoring and predicting rather than about (in)validating the reality of climate change. For addressing the second question, better economic modeling is indispensable. Indeed, Stern conjectures (2013) that a new generation of models is needed in climate science and economics with a stronger focus on lives and livelihoods.

Despite fifteen years of discussions about the Stern report, the two stumbling blocks mentioned in the introduction remain firmly in place. First of all, any small variation in the chosen Ramsey parameters will swamp all other relevant aspects of climate change for future generations, hindering current discussions and decision making. The other stumbling block relates to the intractability of IAMs, an objection already rised by Dasgupta (2007). This thesis addresses both stumbling blocks with an emphasis on the first one.

One attains insight not only by 'getting the numbers right', but also –not to say mainly– by the awareness that the results are to-the-point, intuitively right, and explainable. Or at least that they follow insights from behavioral economics. This level of understanding could evoke new and deeper insights. Although easily formulated, actually reaching deeper insights is not that easy and one could easily be lead astray.

As stated above, novel empirical evidence might affect our thinking of discounting the future. In fact, the discussions about the discount rate for very long periods prompts Giglio et al. (2015) to search for data on independent sentiments toward long term discount rates. They notice that the home ownership in the United Kingdom and in Singapore could give a clue. For this purpose, they compare the prices of houses in the United Kingdom and in Singapore for which the ownership is permanent or temporary. The latter is a leasehold: a prepaid and tradable lease over a very long period, often with maturities between 99 and 999 years. Normal ownership –a freehold– is a perpetual contract. They conclude that households attach significant value to cash flows far in the future.

2.5 Main idea and research questions

Two notions sprout from the Stern report debate. First, discount rates and related numerical concepts are important but controversial ingredients of the search of best (in terms of cost-effective) strategies for climate change mitigation and adaptation. Second, the intractability of climate and economic models (IAMs) could easily hinder decision making. Both notions have incited me to investigate with a small scale and abstract model the influence of a few fundamental financial-economic parameters on externality costs of climate change, called the social cost of carbon.

The first question I address is: Can the Giglio et al. (2015) observations shed new light or solve the controversy between Stern and Nordhaus on the social cost of carbon? More in particular, can a different discount function than the common exponential function be used for long-run discounting?

The second question is: Can a simplified abstract model approximate the social cost of carbon?

The most important financial-economic parameters related to the Stern-Nordhaus controversy are the type of discount function, the rate of pure time preference, the growth rate of the economy, (de-)carbonization of the economy, and the elasticity of the utility function. In this study I will investigate their influences on a fundamental level. Physical or physico-economic parameters, *e.g.* the climate CO_2 sensitivity and the damage-temperature relation, will be taken from the literature as is. The only geophysical parameter that is variable as well is the *natural* CO_2 growth or decay rate, which –as will be shown– has a similar effect on the social cost of carbon as the rate of pure time preference.

Novel empirical evidence could, thus, initiate a paradigm shift in thinking about the value of the future. In this respect, the study by Giglio et al. (2015) provides empirical insights into attitudes of households toward the value of expensive items over very long periods. Obviously, the attitude of households toward the value of residential properties over periods of 100 years or more is a far cry from the value humankind should attribute to a healthy biosphere. Nevertheless, it covers the untrodden middle ground between the market interest rates that dominate Nordhaus' approach and the near-zero rate of pure time preference in the Stern Review.

This study adds to the vast literature on the economic implications of climate change since it not only discusses discounting of externality costs of climate change, but also because it proposes a method of long-run discounting, which smoothly couples opposing but irrefutable viewpoints. In addition, the simplified abstract model developed and analyzed in this study can be applied and extended easily by researchers in the field and by the public without the use of extensive economy-climate (IAM) models.

3. Theory and Methods

3.1 Elevated global temperatures and the economy

Next to labor and capital, energy is a major factor of economic production (Ayres, Van den Bergh, Lindenberger, and Warr, 2013). In order the study the relation between the use of fossil fuels and the subsequent costs for society as an economic externality, one must have a descriptive model for the emission of these gases into the atmosphere, their build up, and the physics and chemistry of the changing atmosphere and its interaction with the broader biosphere, including oceans, soil, vegetation etc. Since the physical behavior of the earth with its atmosphere is not part of the present study, I follow similar approaches as used in most IAMs, in particular the DICE IAM of Nordhaus (Nordhaus, 2013; Nordhaus, 2014).

The emission of greenhouse gases is strongly related to the total gross domestic product (GDP), which depends on the size of the human population and on all its monetizable activities. In the most simple version of my model, I regard a constant growth rate of the GDP, but adaptations, such as a linearly decreasing growth rate, are also possible, see for instance Fig. 7. Furthermore, I assume the population to be constant. The historic growing population is shown by the red dashed curve in Fig. 2, but the forecast by demographers is indeed stabilization near the end of the 21st century[20], likely followed by a long-lasting decline.

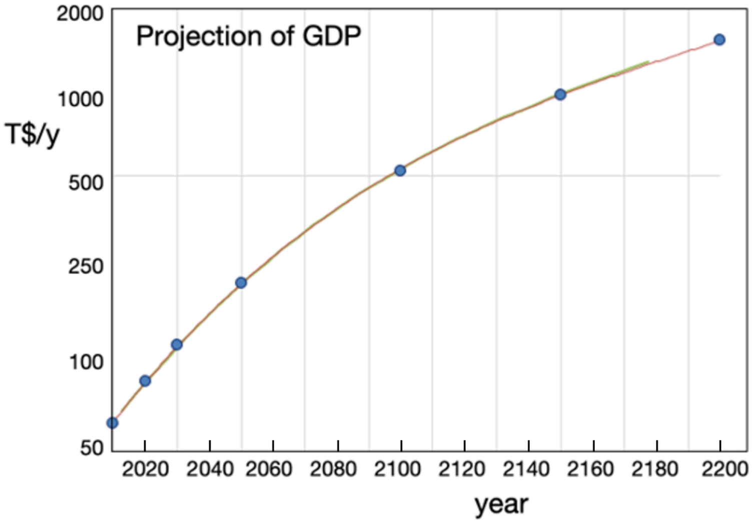

Figure 7. Projection of the GDP until the year 2200. The GDP amounts to 89 T$/y in the reference year 2020. The growth rate drops linearly from 3.1%/y in 2020 to 1.0%/y in 2200. Data points are from Nordhaus (2013).

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[20] E.g.: https://population.un.org/wpp/

Additionally, I assume that the average CO_2 emission per unit of GDP is either constant or changes exponentially with time. Shift in economic activities, more efficient electricity generation, shift from coal to gas, CO_2 storage, and increasing penetration of renewable energy sources drive the CO_2 emission per unit of energy (see Fig. 4b) and per unit of GDP.

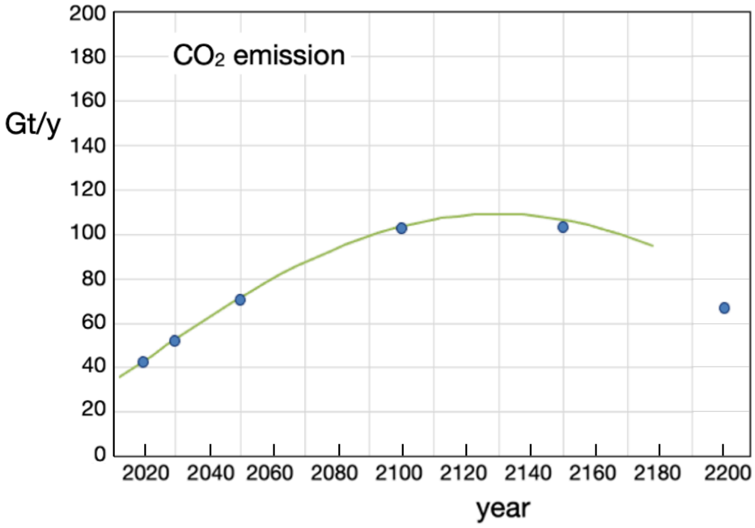

Figure 8. Projected global CO_2 emission for a decay of 1.5%/y in the CO_2 emission per unit of GDP. Data points are from Nordhaus (2013).

Emitted CO_2 does not remain in the atmosphere forever. In fact, the atmospheric residence time of a CO_2 molecule is 100 to 1000s of years (Archer et al., 2009). CO_2 can dissolve into the oceans or taken up via photosynthesis by plants or react chemically with *e.g.* rocks or soil. In my model I assume that the CO_2 level in the atmosphere rises in proportion to the human emission. Natural atmospheric CO_2 emissions that are not related to the economy, are ignored, since they should be equal to their natural removal from the atmosphere. Nevertheless, I do take into account that human-caused CO_2 is removed from the atmosphere by natural processes. Note that many sinks of CO_2 are in fact buffers that might release the absorbed CO_2 again back into the atmosphere. Finally, there are sources of CO_2 linked to other non-energy related processes, *e.g.* in deforestation or cement production, and other types of greenhouse gases, such as CH_4.

An increase in the level of CO_2 –or any other greenhouse gas– reduces the long-wavelength thermal radiation of the earth into space. Part of this radiation is absorbed by atmospheric molecules and, subsequently, re-emitted into a random direction, also back to the earth' surface. Because of the increasing CO_2 level (Fig. 3), the long-wavelength thermal radiation absorption and subsequent re-emission are increasing. In other words, the earth surface is becoming increasingly irradiated with long-wavelength radiation, leading to an increasing heating, see Fig. 5. The extra irradiation received by the earth surface from the anthropogenic greenhouse gases is called extra radiative forcing, at present about 2 W/m^2.[21] A higher surface temperature leads to increased thermal radiation. This process eventually stops the extra heating by an enhanced level of greenhouse gases and a new balance between thermal radiation emission and absorption is established.

In general, a logarithmic relationship between atmospheric CO_2 concentration L at time t and increased temperature T is assumed (Cline, 1992):

$$T(t) = \lambda\, F_0 \ln\left(\frac{L(t)}{L_p}\right) \quad , \qquad \text{(Eq. 2)}$$

where λ is the climate sensitivity (0.8 °C.m^2/W), F_0 is the radiative forcing constant (5.35 W/m^2) and L_p is the pre-industrial atmospheric level of CO_2: 278 ppm. T is the global mean temperature above the temperature at the start of the industrial revolution, taken to be 1750; t is time. The warming of the earth is, however, a dynamic process. There is a lag between the actual temperature rise and the final equilibrium one, if that ever will be attained. Moreover, the logarithmic dependence is uncertain (Cline,1992). As long as the increase in CO_2 level $L(t)–L_p$ is less than L_p, the logarithmic dependence is close to linear. At present (2020), $L(t)$ is just above 400 ppm, almost 50% higher than in 1750.

The relation between increased global temperature and damage to the economy (the GDP) is very uncertain (Burke, Hsiang, and Miguel, 2015). It is even not clear what could be called damage and how it can be measured. This discussion is, however, outside the scope of this study. With Nordhaus and others (Nordhaus, 2013; Nordhaus, 2017) I assume that the damage is proportional to the square of the elevated temperature, as quantified by Tol (2009):

$$D_r(T) = \varphi_2\, T^2 \quad , \qquad \text{(Eq. 3)}$$

with φ_2 equal to 0.00236 /°C^2, see Fig. 9a. Howard & Sterner (2017), however, take φ_2 equal to 0.01004 /°C^2, whereas Burke et al. (2015) find a much stronger relationship, with damages over 25% of the global GDP in 2100 at an increased temperature of 3 °C, see Fig. 9b.

The absolute damage D in T\$/y [22] is given by:

$$D(t) = D_r\big(T(t)\big)\,.\,G(t) \quad , \qquad \text{(Eq. 4)}$$

where $G(t)$ denotes the size (GDP) of the global economy at time t. Note that, since I regard population as constant, GDP and GDP/capita are equivalent, apart from a numerical constant.

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[21] Compare: average radiation received from the solar (short-wavelength) radiation is 340 W/m^2. The human energy use is 0.03 W/m^2.

[22] 1 T\$ = 1,000,000,000,000 \$, 1 trillion dollars or 1 Teradollar.

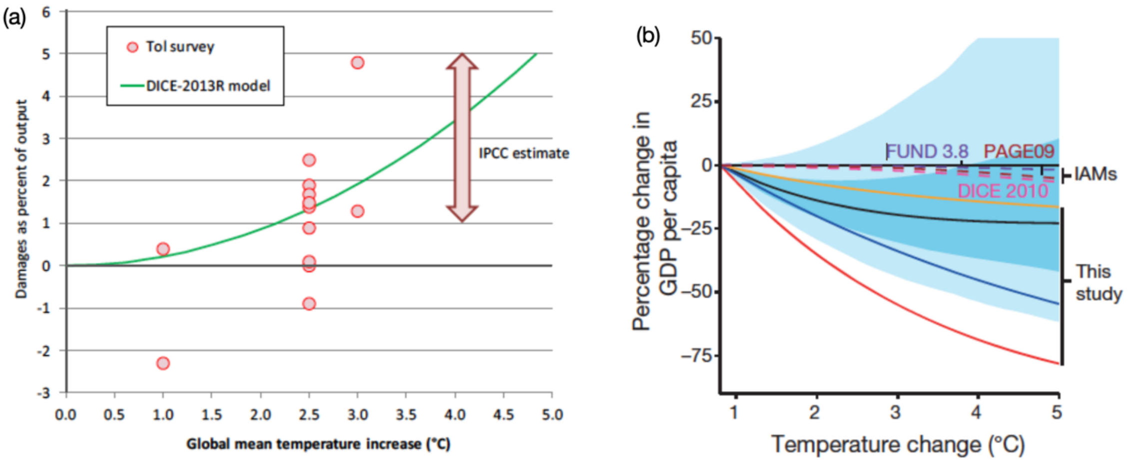

Figure 9. Estimates of the impact of climate change on the economy. (a) From Nordhaus (2013, 2017) and Tol (2009); (b) From Burke et al. (2015). The green parabola in (a) is the function of Eq. 3, adopted in the present paper. The damage at 3°C in (a) is 2%, whereas in (b) it ranges between 10% and 55%.

With equations 1 to 4 I have the ingredients in my model to calculate the future damage to the economy by the emission of one unit of CO_2 today. The only remaining factor is the method of discounting future costs caused by the damage. For this purpose, I use a discount function $f(t)$, which is 1 for $t = 0$ (today) and decays to 0 for $t \rightarrow \infty$.

3.2 Economically rational and behavioral discounting

Future profits and costs are usually valued differently than current ones (Gollier et al., 2008). In 1928, Frank Ramsey introduces the concept of impatient but rational economic agents, who value future utility less than current utility. However, the degree to which future utility has less value ('is discounted') is not a forgone conclusion. In general, but not necessarily, the discount increases monotonically[23] with time into the future until it reaches 100% and the remaining present value is zero. Under certain circumstances, one can quantify the discount theoretically. Consider *e.g.* an agent who has investment and consumption opportunities depicted by the graphs of Fig. 10.

Assume that the agent is rational but impatient and has the possibility at time 0 to transfer (*i.e.* invest[24]) a sum of $-A$ in a 'blue' project, which will transfer (*i.e.* return) to hir at time τ the

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[23] The monotonically is not absolute. In July, for instance, the promise of the use of a winter coat 6 months later might have more value than the offer to use it today.

[24] Note that a negative transfer of money by an agent is either an investment or a consumption; a positive transfer is a return.

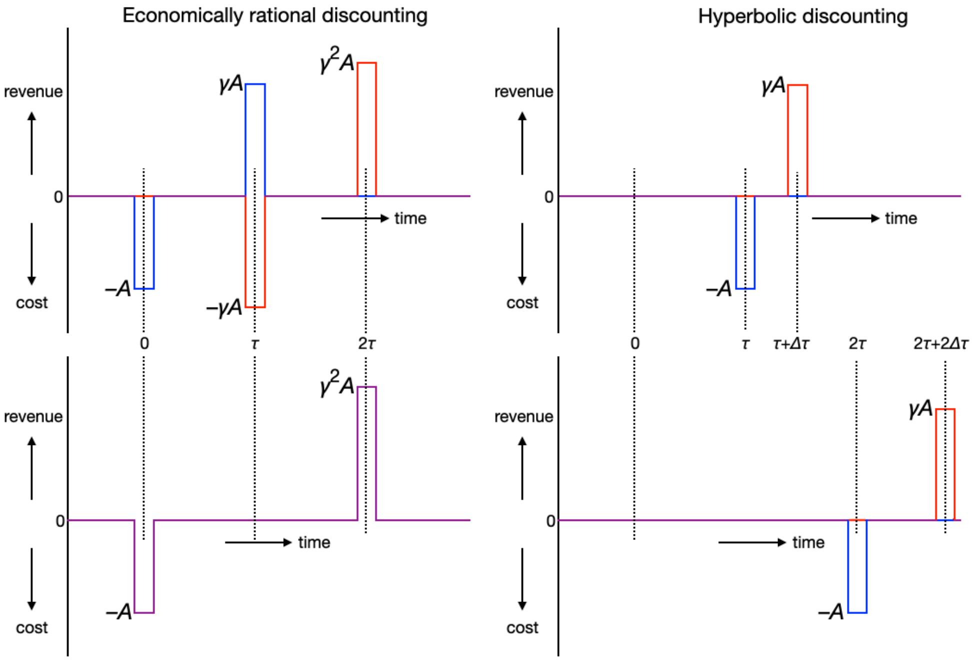

Figure 10. Economically rational discounting (left) and behavioral hyperbolic discounting (right).

sum of γA (>A), see the left graph. Assume also that the agent considers this particular return just sufficient to offset the discomfort of postponing consumption of A at time 0 to time τ. Thus, the agent discounts at time 0 the γ times higher consumption at the future time τ by a factor of $1/\gamma$. Furthermore, at time τ the agent could either consume the return or invest it in a new (red) project, with the same relative return factor γ. Then, at time 2τ the agent could enjoy the consumption of $\gamma^2 A$. Assume furthermore that all circumstances remain equal between time 0 and time 2τ and that no new relevant information will arrive in this time period. Moreover, assume that investing $-\gamma A$ and receiving $\gamma^2 A$ a time later has a γ times higher utility than investing $-A$ and receiving γA a same time later. Then the agent could equally well consider at time 0 a project with an investment of $-A$ at time 0 and a return of $\gamma^2 A$ at time 2τ. Because the cash flows are equal, the alternative program is equally useful to hir. Hence, the agent discounts the revenue of $\gamma^2 A$ at time 2τ by a (discount) factor of $1/\gamma^2$. By continued repetition of this argument, the agent discounts at time 0 all revenues at time $x\tau$ by a discount factor f of $1/\gamma^x$.

With $t \equiv x\tau$, one can write:

$$f(t) = \frac{1}{\gamma^{\frac{t}{\tau}}} = \exp\left(-\frac{t}{\tau}\ln\gamma\right) = \exp(-st) \qquad , \qquad \text{(Eq. 5a)}$$

where $s \equiv \frac{\ln\gamma}{\tau}$. (Eq. 5b)

This is exponential discounting, used abundantly in finance to link the value of transactions settled at different points in time. Important is to realize that in the entire time span considered, circumstances must remain equal and no new information should arrive that can affect financial or economic decisions. In principle, the discount rate parameter s depends on the level of impatience of the agent. However, if the agent could always borrow or lend money, then the discount rate becomes the actual interest rate.

However, if circumstances are not constant or predictable and if the impatience cannot be accommodated financially, humans apply different discount factors to future events, depending *e.g.* on the size and the delay of the event (Thaler, 1981). Some people who are given the choice to get one apple today or two tomorrow, will opt for a single apple today, while others prefer to wait one day (Thaler, 1981). However, if they are given the choice to consume one apple exactly one year from now or two apples after one year plus one day, almost all prefer to wait the last 366th (or 367th) day as well, thus regarding the extra 0.3% waiting time a trifle. For them, the *additional* discount, when the enjoyment of a utility is postponed from time τ to time $\tau+\Delta\tau$, depends on the relative delay $\Delta\tau/\tau$. The psychological attitude toward future time is described in the scientific literature as a logarithmic representation of time (see *e.g.* Jozefowiez, Staddon, and Cerutti, 2009). For economic agents, who can lend and borrow and know that circumstances and information will not change, the *additional* discount, when options are postponed from time τ to time $\tau+\Delta\tau$, depend on $\Delta\tau$ only, not on τ, and hence can be discounted by Eq. 5a.

Because $\frac{\Delta\tau}{\tau} = \Delta \ln \tau$, one could replace t in Eq. 5a by $\ln t$ and write the behavioral discount factor as[25]:

$$f_p(t) = \exp\left(-w \ln t\right) = t^{-w} \quad . \qquad \text{(Eq. 6a)}$$

Or, in order to avoid a singularity at $t = 0$ and to ensure $f_p(0) = 1$,

$$f_p(t) = \frac{1}{(1+\frac{t}{t_0})^w} \quad . \qquad \text{(Eq. 6b)}$$

Equation 6b describes a power-law discount function. If $\alpha << w^{-1}$

$$f_p(t(1+\alpha)) = \frac{1}{(1+\frac{t(1+\alpha)}{t_0})^w} = \frac{1}{(\frac{1}{1+\alpha}+\frac{t}{t_0})^w} . \frac{1}{(1+\alpha)^w} \approx \frac{1}{\left(1+\frac{t}{t_0}\right)^w} . (1 - \mathrm{w}\alpha) = (1 - \mathrm{w}\alpha) . f_p(t) \quad . \qquad \text{(Eq. 6c)} .$$

Thus, if the waiting time changes by 1%, the discount factor changes by $-w$%. For $w = 1$, one gets the hyperbolic discounting (Rubinstein, 2003). Hyperbolic discounting is being regarded as time

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[25] This is a hand waiving proof, not a rigorous mathematical one.

inconsistent (Strotz, 1955; Frederick, Loewenstein, & O'Donoghue, 2002), but also better in describing human behavior or preferences in case of uncertain outcomes (Green & Myerson, 2004). In the apple example, hyperbolic discounting implies that after one year some individuals who expressed their willingness to wait one year plus one day, will eventually change their mind and opt for the single apple, one day earlier. Hence, their choice is not consistent in the passing of time.

3.3 Long-run discount study by Giglio et al.

The hedonic regression results for the UK data in the study by Giglio et al. (2015) combined with estimates of the average expected rate of return to housing provide information about the term structure household apply for long-term discounting. By regression analysis of almost 1.4 million prices of freehold and leasehold houses, the authors find that leaseholds with 80-99 years maturity sell for 17% less than similar freeholds, whereas leaseholds between 150 and 300 years and longer sell for 3.5% less. They conclude that for a 100-year maturity, the discount rate is 2.6%/y, being 1.9%/y derived from the regression analysis plus 0.7%/y for the estimated (real) growth rate of housing rents. The authors decompose the resulting 2.6%/y in a 1.0%/y risk-free (real) discount rate plus a risk premium of 1.6%/y. Obviously, longer or shorter maturities could have different discount rates, but the authors do not discuss them. The aim of my study is to fit Giglio's observations with a power-law function in order to link the discounting applied by households to a behavioral discount model. Note that Giglio et al. (2015) do not follow this approach.

3.4 Reference energy-climate-economy model

My reference energy-climate-economy model is based on the standard climate model plus a simplified abstract economy-energy model based on two parameters: GDP growth rate g and (de-)carbonization rate c. Here, the latter refers to the (de-)carbonization of the economy, *i.e.* the growth ($c > 0$) or reduction ($c < 0$) in CO_2 emission per unit of GDP. Whether the decrease is due to a shift from high-to-low carbon fuels, energy savings, shift to renewable energy sources, or from CO_2-emission to -capture, is not relevant. In the standard use, the model assumes constant rates. To the standard climate model, described in the previous chapter, I add only one parameter: the *natural* decay rate h of *human-caused* CO_2. It is noted that I make no attempt to optimize consumption and investment, as is often done in Integrated Assessment Models; thus, consumption and economic output are being regarded as identical at all times.

In order to investigate dependencies, the parameters g, c, and h can have three values: small, medium and large. The long-term global GDP growth amounts to 1 – 2%/y (Nordhaus, 2017). In the coming decades a decrease in g of about 2 – 1%/y is foreseen. Although the GDP per capita is expected to grow longer, the anticipated decrease in population after 2050 will reduce total GDP growth. Therefore, I take 1%/y as the high value. There is a long debate about the possibility of perpetual economic growth. Planetary boundaries (Raworth, 2017, p. 44) could force GDP growth to zero this century. Whether that will indeed happen or not, is highly uncertain. Nevertheless, I take 0%/y is the lower boundary for g.

The present rate of (de-)carbonization is negative, varying between –1.5%/y and 0.5%/y in the last decades. (See also Fig. 4a.) However, since the actual carbon-based fraction of energy use is still close to 90%, it is not guaranteed that decarbonization will indeed proceed. Therefore, I take 0%/y is the upper value for the (de-)carbonization rate c. As largest negative value, I take –1%/y. The natural decay of human-caused CO_2 is not a well-defined physical process, but should be taken into account in economic damage models of pollution (Eismont, 1994). Therefore, I take as upper boundary 0%/y (implying in fact that it does not exist). In Archer et al. (2009) a CO_2-residence time in the atmosphere of between one hundred and thousands of years is mentioned, corresponding to a decay rate h between –1%/y and –0.03 %/y. Van den Bijgaart Gerlagh, and Liski (2016) estimate h (called $-\delta_S$) to lie between –2%/y and –1%/y. Two important sinks of CO_2 are dissolution into the oceans and chemical reaction with soils and rocks. However, the uptake of CO_2 by these buffers could later lead to new release and, hence, change from sink to source. These natural exchange processes complicate the fate of emitted quantities CO_2. In this work, I neglect the effects of exchange between buffers. Moreover, I ignore other greenhouse gases.

In sum, I take for the minimum values of all three absolute rates ($|g|$, $|c|$ and $|h|$) 0%/y, as maximum 1%/y, and as medium 0.5%/y. These values cover a realistic range and incidentally show nice symmetry.

Furthermore, I take a long time span of the model: 1000 years. In this way, slow trends and saturation effects will more easily discernible. Note that a 1%/y growth over 1000 years implies an amplification factor of $e^{10} \sim 22{,}000$; at 0.5%/y, it is 150. Starting values of the variables and relevant parameters are summarized in Table 1. It is noted that in these reference calculations only exponential discounting is used.

Although this model highly abstract and simplified, thus barely realistic, it will reveal some important fundamental relationships. The entire list is in appendix 1. Appendix 2 shows a part of an example calculation of some of the excel version of the model.

Table 1: Summary of the most important input and output variables, parameters and functions.

Variable	remark or alternative name	symbol	lower bound 1)	upper bound 1)	Nordhaus 2017 7)	unit
present			2020		2010	y
year	year 0 = is present	t	0	1000	0...1000 8)	year or y
GDP growth rate	GDP per capita growth rate	g	0	1.5	2.1...0.0 8)	%/y
(de-) carbonization rate	change in global CO_2 emission / GDP	c	–1.5	0	–1.5	%/y
C	growth rate in carbon use ($= g + c$)	C	–1	1	var.	%/y
natural CO_2 growth rate	($-\delta$ or $-\delta_S$ in the literature)	h	–1	0	–0.5 / 0 9)	%/y
discount rate	(if exponential)	s	$g + h$	$g + h + 4$	4.25	%/y
rate of pure time preference	growth-corrected discount rate in Nordhaus	ρ ($= s - g$)	$-h$	4	1.42 / 2.55 9)	%/y
equality parameter	also called risk-avoidance parameter	η	1	1.5	1	
emission	start CO_2 emission	E_0	56	4)	33	Gt/y
size of economy	start size global economy, GDP	G_0	80	4)	66	T$/y
	pre-industrial atmospheric CO_2 level	L_p	278	4)	278	ppm
	start atmospheric CO_2 level	L_0	400	4)	380	ppm
elevated temperature	start global temperature above pre-industrial value	T_0	1.56 5)	4)	1.35 5)	°C
	linear damage coefficient	φ_1	0	4)	4)	% . °C^{-1}
	quadratic damage coefficient	φ_2	0.236	4)	0.236	% . °C^{-2}

1) These are typical values, exceptions are possible.

2) For calculation purposes

3) For plotting purposes

4) No variation applied, identical to lower bound

5) Because of neglected lags, the model value is higher than the actually observed temperature

6) Output variable

7) Comparison calculation with Nordhaus (2017)

8) Range in each calculation

9) In various calculations

4. Results and Interpretations

4.1 Analysis of long-term discount factors

The red circles in Figure 11 show the observations of Giglio et al. (2015) of the discount factor that UK households apply to future value of houses. Red curves are three fitted functions. The full curve is a power-law function of the form

$$f_p(t) = \frac{1}{(1+t/t_0)^w} \quad . \tag{Eq. 7a}$$

Best fit is obtained for w=2.0 (±0.1) and t_0 = 56 (±2) y. Note that the data point at 880 y has large uncertainty.

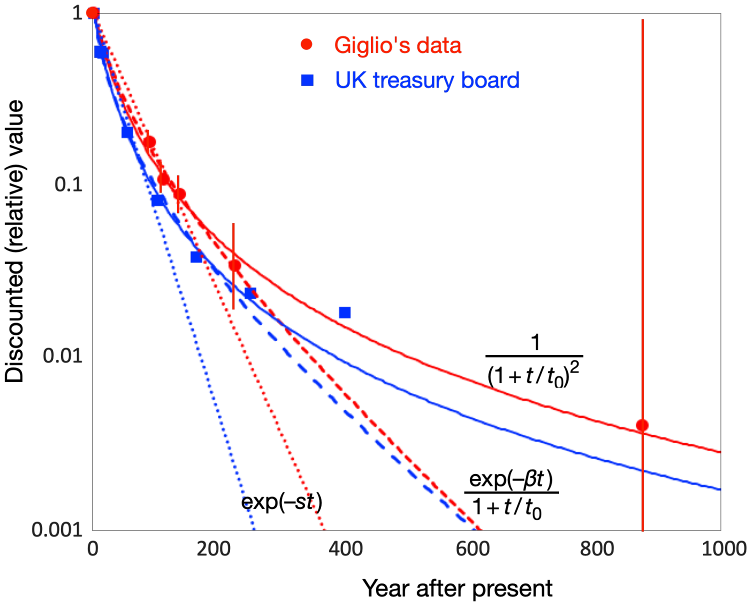

Figure 11. The discount factor in the observations by Giglio et al. (2015) of UK freehold and leasehold sales (red circles with 2σ-error bars) plus a power-law fit (red full curve). The blue squares represent the term structure of interest rates with long maturities recommended by the UK (HM Treasure, 2003); the blue curve is a power-law fit of power 2. The red dotted straight line fits the four Giglio data points between 80 and 250 years; the blue one fits the UK Treasury data between 15 and 125 years. The dashed curves combine an exponential plus a power-law function with power 1.

The dashed curve a combined exponential plus power-law function:

$$f_c(t) = \frac{\exp(-\beta t)}{(1+t/t_0)^w} \quad . \tag{Eq. 7b}$$

For w = 1, the fit parameters are β = 0.66%/y and t_0 = 38 (±2) y. (Note that I excluded here the data point at 880 y.) A combined exponential-power-law function is proposed recently by

Giglio.[26] Finally, the dotted red curve is an exponential fit of the three data points between 80 and 150 years. The decay rate is –1.9%/y.

The data of Giglio et al. (2015) describe observed long-term discount rates. A series of proscribed discount rates is available as well. Table 2 shows the term structure of interest rates, as recommended by the UK Treasury Board (HM Treasury, 2003). The discount factors in the last column are depicted as the blue squares in Fig. 11.

Table 2. Term structure of discount rates r_d with various maturities, as recommended by the HM Treasury (2003). The discount factor is calculated as $DF = \exp\,(-M_c.r_d)$, where M_c is the central maturity in its class.

Maturity (y)	Discount rate, r_d (%/y)	Central maturity, M_c (y)	Discount factor, *DF*
<30	3.5	15	0.5916
31 – 75	3.0	53	0.2039
76 – 125	2.5	100	0.0821
126 – 200	2.0	163	0.0384
201 – 300	1.5	250	0.0235
301	1.0	400	0.0067

Also the discount factors of the UK Treasury Board are fitted with the functions of Eqs. 7a and 7b, see the blue data points and curves in Fig. 11. Table 3 summarized all fit results. The last line gives the slopes of the discount function at t = 0, equal to w/t_0, $1/\ t_0 + \beta$, and r_d for the power-law, the combined, and the exponential function, respectively.

Both the Gilgio data and the UK Treasury Board data seem to follow (combined) power-law discount functions. The initial parts of the (combined) power-law discount functions (*i.e* the slopes at $t = 0$) have values, not very different from usual market interest rates. Of course, since the uncertainties are quite large, these results do not prove that individuals do apply these discount functions. Nevertheless, it is clear that a fixed discount rate is not applicable for discounting simultaneously over short ànd long periods.

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[26] https://www.youtube.com/watch?v=7CpTAaSl6jI

Table 3. Fit results for the Giglio et al. (2015) observations and the HM Treasury (2003) recommendations for three different discount functions.

	Giglio			UK Treasury		
	power-law	combined[1]	exponential[2]	power-law	combined[1]	exponential[2]
w	2.0 (0.1)	1 [3]	–	2.0 (0.1)	(1)	–
t_0 (y)	56 (2)	38 (2)	–	43 (2)	21 (2)	–
β, r_d (%/y)	–	0.66 (0.03)	1.9 (0.1)	–	0.58 (0.03)	?
slope at *t* = 0 *(%/y)*	3.6 (0.2)	3.3 (0.2)	1.9 (0.1)	4.7 (0.2)	5.3 (0.2)	

1) Data point at 880 y ignored
2) Only data points < 150 y
3) Not a fit parameter

The values in Table 3 will be used later to estimate the SCC over a time span of 1000 y.

4.2 Reference calculations

Reference calculations are performed in order to give insight into fundamental relationships. Figure 12 shows the outcome of nine reference calculations with variable GDP growth rate *g,* (de-)carbonization rate *c,* and natural carbon growth rate *h*. In all calculations, the population is constant. When $g = c = h = 0$%/y (Fig. 12a), the CO_2 emissions are constant (56 Gt/y), but the atmospheric CO_2 level (purple curve) rises continuously, so do the temperature (light green) and the damage to the GDP (brown). The elevated temperature surpasses 10 °C 680 years after present (AP), when the relative damage to the GDP amounts to 24%.

In Fig. 12b, the CO_2 emission drops ($c = –1$%/y), the economy does not grow ($g = 0$%/y), and atmospheric CO_2 disappears by natural processes ($h = –0.5$%). As a consequently, the CO_2 level and the temperature reach after 100 y their maximum of 530 ppm and 2.7 °C, respectively. Subsequently, they drop slowly to the pre-industrial levels. The situation in Fig. 12c is similar to that in Fig. 12a, but here the economy continues to grow. Note that the extra information given in (c) applies to all sub-figures. Because of decarbonization ($c = –g$), the CO_2 emissions remain constant. Therefore, their CO_2 levels and temperatures are equal, but the absolute damage to the economy is of course larger in (c), as compared to (a). In all cases with the same C (= $c + g$) and the same *h*, the curves of the CO_2 level *L*, the temperature *T* and relative GDP damage D_r are equal.; compare *e.g.* (a) with (c) and (b) with (f).

In Fig. 12d a constant amount of CO_2 is emitted per unit of GDP (c = 0%/y), while the economy grows moderately but steadily at 0.5%/y. In year 900 AP the temperature increase reaches 21 °C and the relative damage to the GDP becomes 100%, a rather absurd situation that would have led to a devastating catastrophe already many centuries earlier. Nevertheless, the model calculation is sound.

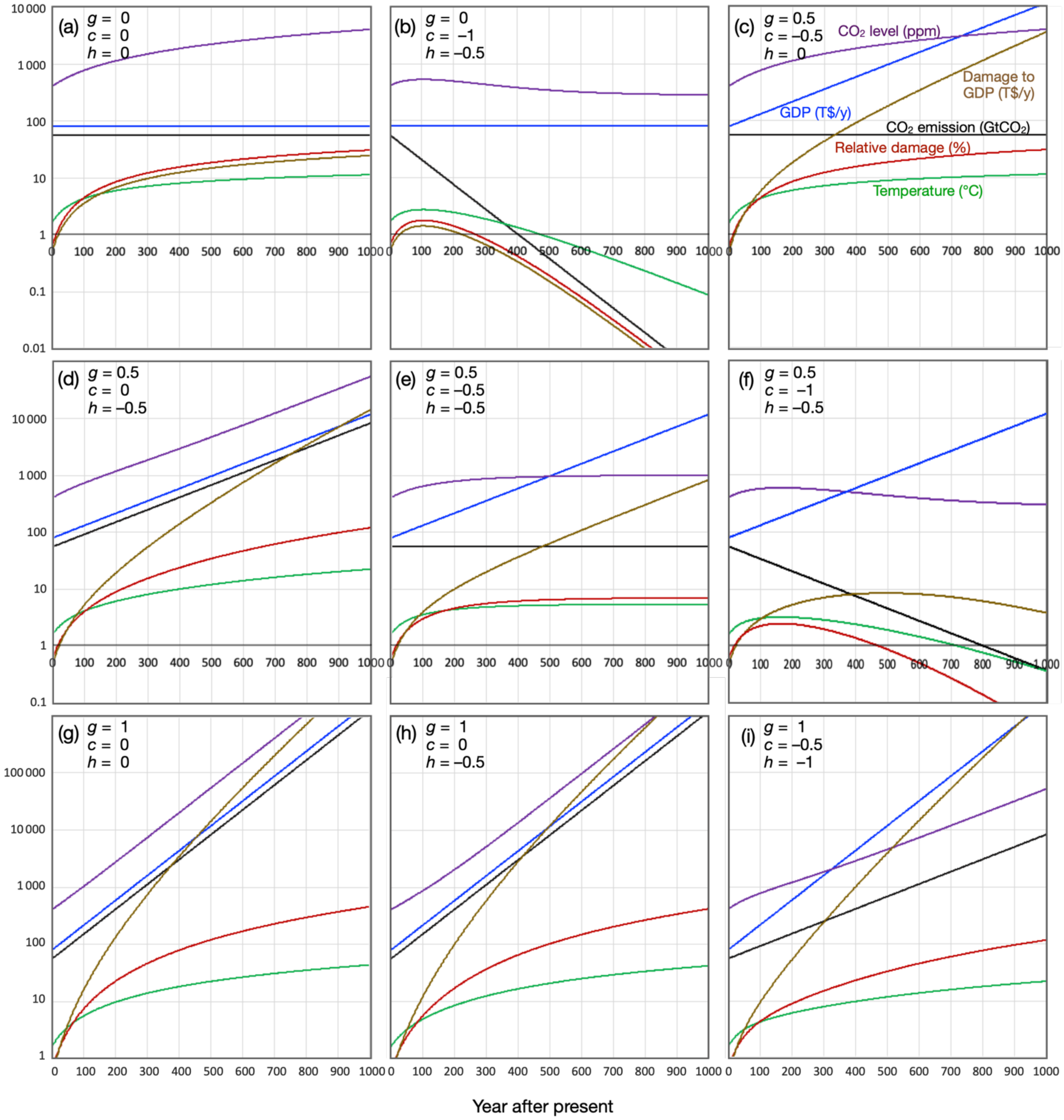

Figure 12. Evolution in the simplified climate-economy-energy model for a selection of economy/climate parameters. In (c), the curves are labeled and corresponding units are given. g is the growth rate of the economy (GDP); c is the (de-)carbonization rate of the economy; the growth or decay by natural processes of human CO_2 emission occurs at rate h (g, c and h are expressed in %/y).

In contrast, if CO_2 emissions are kept constant by sufficient decarbonization of the economy ($c = -g$) (e), the CO_2 level saturates in year 300 near 1000 ppm. Then, the annual emission equals the natural decay and the temperature saturates at 5.5 °C above the pre-industrial value. With 1%/y natural decay (not shown), the final CO_2 level is 640 ppm with a final elevated temperature of 3.6 °C, already almost reached in year 200 AP.

The situation changes dramatically with centuries of high economic growth, as shown in the lower panel of Fig. 12. Cases (g) and (h) have the same g and c, but different h. However, because the emissions grow so rapidly ($C = c + g = 1$%/y), the natural decay has a negligible influence on the rapidly growing CO_2 level, the temperature and the GDP damage. Note that the strong growth in CO_2 emission causes full collapse of the economy (*i.e.* $D_r = 100$%) in the second half of the 5th century.

All nine outcomes of Fig. 12 plus 18 not-shown outcomes are summarized in table 4.

Table 4. Summary of the most important variables in the reference energy-climate-economy model. T_m is the maximum temperature in the investigated 1000-year period and t_m is the year of the maximum. T_f is the final temperature of that period and dT_f/dt is the change in temperature in the final year 1000; c, g and h are expressed in %/y. The dark orange box at the upper right is the situation with extreme temperatures. Red values are final or maximal temperatures between 10 and 30 °C; black values between 4 and 10 °C; green values are between 0 and 4 °C. (The ordering in the table is such that the highest temperatures are found in the upper right corner and the lowest in the lower left corner.)

		g = 0			*g* = 0.5			*g* = 1			
		h=–1	*h*=–0.5	*h*=0	*h*=–1	*h*=–0.5	*h*=0	*h*=–1	*h*=–0.5	*h*=0	*units*
c = 0	T_m	3.6	5.5	11.5	20.8	22.6	25.5	41.0	42.2	44.0	°C
	t_m	1000	1000	1000	1000	1000	1000	1000	1000	1000	y
	T_f	3.6	5.5	11.5	20.8	22.6	25.5	41.0	42.2	44.0	°C
	dT_f/dt	0.0	0.0	0.0	0.1	0.1	0.1	0.1	0.1	0.1	10^{-3} °C/y
c = –0.5	T_m	2.5	3.3	6.0	3.6	5.5	11.5	20.8	22.6	25.5	°C
	t_m	101	164	1000	1000	1000	1000	1000	1000	1000	y
	T_f	0.1	0.4	6.0	3.6	5.5	11.5	20.8	22.6	25.5	°C
	dT_f/dt	–0.5	–0.4	0.0	0.0	0.0	0.0	0.1	0.1	0.1	10^{-3} °C/y
c = –1	T_m	2.2	2.7	4.3	2.5	3.3	6.0	3.6	5.5	11.5	°C
	t_m	64	106	1000	101	164	1000	1000	1000	1000	y
	T_f	0.3	0.1	4.3	0.1	0.4	6.0	3.6	5.5	11.5	°C
	dT_f/dt	–0.9	–0.5	0.0	–0.5	–0.4	0.0	0.0	0.0	0.0	10^{-3} °C/y

The table lists for low, medium and high negative values of the CO_2 natural decay rate *h,* of the (de-)carbonization rate c and of the GDP growth rate g the maximum temperature T_m in the entire 1000-year period, plus the year y_m of the maximum. If the temperature continues to rise, the maximum occurs of course in year 1000. It lists also the derivative of the temperature dT_f /dt in the final year 1000; a positive value indicating a continuous increase, a negative value a drop, 0 meaning saturation. Note that the outcomes are color-coded. Green numbers indicate a maximum of 4.0 °C or less, a situation with limited damage to the biosphere. Black numbers for intermediate maximums between 4 °C and 10 °C, whereas red numbers refer to high maximums between 10 °C and 30 °C. Finally, the red boxes indicate extreme temperatures above 30 °C. The lower temperatures are in the lower left corner of the table, where both the natural decay h and the human decarbonization rate c are highly negative. The extremely high temperatures are in the upper right corner. Note that an economic growth of 1%/y requires that both h and c are –1%/y to avoid elevated global temperatures above 4 °C; in simple formulation: $|c+h| > 2g$. Note also the high symmetry in the table: all diagonals with the same value for $c - g$ have identical contents. Thus, there is room for economic growth, as long as it goes along with strong reduction of fossil fuel use and/or CO_2 removal. Another interesting observation is that at low g (left columns) or high $-c$ (lowest rows), the effect of 0.5%/y extra natural decay implies a halving of the final temperature rise. In contrast, if g is larger than c, natural decay has a negligible effect. Apparently, there is no recourse against strong economic growth plus weak decarbonization.

Although these stylized calculations can hardly be called realistic, miss many important details, and unlikely lead to major new insights in climate change, they serve as background for coming calculations. Moreover, they help us to concentrate on the fundamental parameters and relations and, hence, support understanding of socio-economic consequences of climate change and related abatement policies. In the next section I will calculate the marginal present-value (PV) cost per ton of CO_2 emitted into the atmosphere, *i.e.* the social cost of carbon, SCC. Alongside to the natural decay rate and the GDP growth and decarbonization rate, I will add the socio-economic discount function. It will be shown that symmetries in the results will lead to a simple formula for SCC.

4.3 Calculation of the marginal social cost of CO_2 emission

With the reference models of the previous section, it is rather simple to calculate the marginal costs of CO_2 emission at time 0.

Figure 13b+c shows the curves of the GDP, the CO_2 emission and level, the temperature, and the damage to the GDP. Note that Fig. 13b is identical to Fig. 12e. In addition, the figure shows the PV of the damage, either calculated with the power-law function (PV_p, red full curve) or the exponential function (PV_e, dashed red curve). After a model calculation, the calculation is repeated with the same conditions, but with an extra Γ ton of CO_2 added to the normal emission in year 0. Note that no effort is made to optimize the new emission path, as is done by Nordhaus (2017), though with marginal effects only. The two calculations, thus the one without and the one with extra CO_2 in year 0, are shown as dotted curves in Fig. 13a (note that input parameters are not equal to those in Fig. 13b+c). The difference of the PV curves in the original and in the modified calculation –full curves in (a)–, divided by Γ, is the differential SCC(t). The difference curves SCC_p and SCC_e in Fig. 13 are drawn in dark green.

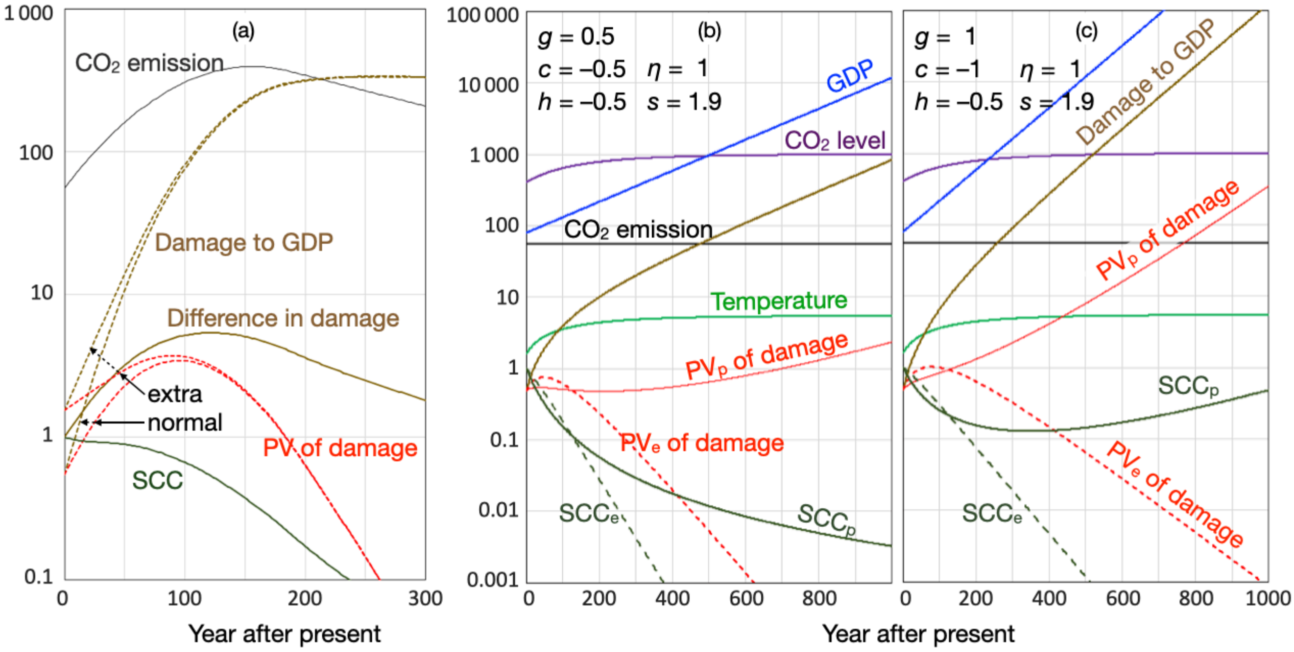

Figure 13. Time-evolution of the GDP (blue, in T\$/y), the CO_2 emission (gray, in Gt/y), absolute climate damage to the economy (GDP) (brown in T\$/y), discounted damage (red, in T\$/y), and the marginal discounted damage (dark green). In (a) the dotted curves are two calculations without (lower) and with (upper) an extra emission in year 0 of 100 times the normal emission in that year; their differences are the full brown and green curves.

In order to make the modification visible in (a), a very large Γ is chosen: 100 times the emission E_0 in the starting year. Of course, since $\Gamma > 0$, the modified curves lie above the original ones, though the differences disappear after about 100 years.

The marginal cost C_m is given by:

$$C_m(\Gamma) = \frac{1}{\Gamma}\int_0^\infty D_\Gamma(t) - D_0(t)\,dt$$

$$= \frac{1}{\Gamma}\int_0^\infty D(T_\Gamma(t)) - D(T_0(t))\ dt$$

$$= \frac{1}{\Gamma}\int_0^\infty D(T(L_{C,\Gamma}(t))) - D(T(L_{C,0}(t)))\ dt \qquad \text{(Eq. 8a)}$$

and the present value of the marginal costs is:

$$C_{PV}(\Gamma) = \frac{1}{\Gamma} \int_0^\infty f(t).\,(D_\Gamma(t) - D_0(t))\,dt \qquad . \qquad \text{(Eq. 8b)}$$

Here, $D_0(t)$ is the original damage function and $D_\Gamma(t)$ the one after the extra emission of Γ in year 0. Furthermore, $f(t)$ is the discount function. In Fig. 13a, an exponential discount function is chosen. The present value of the marginal costs in the limit $\Gamma \rightarrow 0$ is per definition the social cost of carbon, SCC:

$$\mathrm{SCC} \equiv C_{PV}(\Gamma)_{\Gamma\to 0} = (\frac{1}{\Gamma} \int_0^\infty f(t).\,(D_\Gamma(t) - D_0(t))\,dt)_{\Gamma\to 0} \qquad \text{(Eq. 8c)}$$

$$\text{or } \mathrm{SCC} = \int_0^\infty \mathrm{SCC}(t)\,dt \qquad . \qquad \text{(Eq. 8d)}$$

In practical calculations, a small finite value for Γ suffices, *e.g.* E_0. Thus, the integral of the SCC curves in Fig. 13 gives the central number of this thesis: the social (discounted) cost of carbon, SCC. Although the integral should be taken up to infinity and although the damage might persist long or even continue to increase, the present value of the marginal costs will in many cases drop sufficiently fast to 0, although not in all cases.

The growth rates g of the economy and the (de-)carbonization rates c in Fig 13b and 13c differ. But equal amounts of CO_2 are being emitted. The climate behaves similarly. Since the same discount rates s (of 1.9%/y) is used for both cases, their $\mathrm{SCC_e}$ differ. If, however, the same ρ (= $s - g$) are used, exactly the same $\mathrm{SCC_e}$ are obtained.

Power-law discounting poses a problem here: eventually the $\mathrm{SCC_p}$ curve curls upward and therefore its integral is not well defined. This holds especially for Fig. 13c, where g = 1%/y.

4.4 Dependence of SCC on rate of pure time preference

In the previous section the marginal economic damage of present CO_2 emissions in the coming centuries is calculated for a number of stylized growth rates. In this section the dependencies of the social cost of carbon SCC are studied. Obviously, the choice of the discount rate is central here. In this section, only exponential discounting is being applied, in line with most other studies.

Figure 14 shows the most important results. Purple, green and red data points are the SCC as a function of the rate of pure time preference ρ, calculated for natural CO_2 growth rates h between –1 and 0%/y. The top figure (a) shows the inverse of the SCC, the bottom one (b) the actual SCC. For h = –0.5%/y (green circles), five data points are plotted per ρ value, each one for a different C value between –1%/y and +1%/y. For the purple h = –1%/y and the red h = 0%/y data, C is either 0%/y or 0.5%/y. The C = 0%/y data point is always the lowest of its group in (a), and the highest in (b). Through all $C = 0\%/y$ data points in (a) linear fits are made. Their intercepts with the x-axis are –0.92, –0.43 and +0.00%/y; 1-σ uncertainty is 0.02%/y. Their slopes are 0.0210, 0.0201 and 0.0190 tCO_2.y/($.%), respectively; 1-$\sigma$ uncertainty is 0.0004 tCO_2.y/($.%).

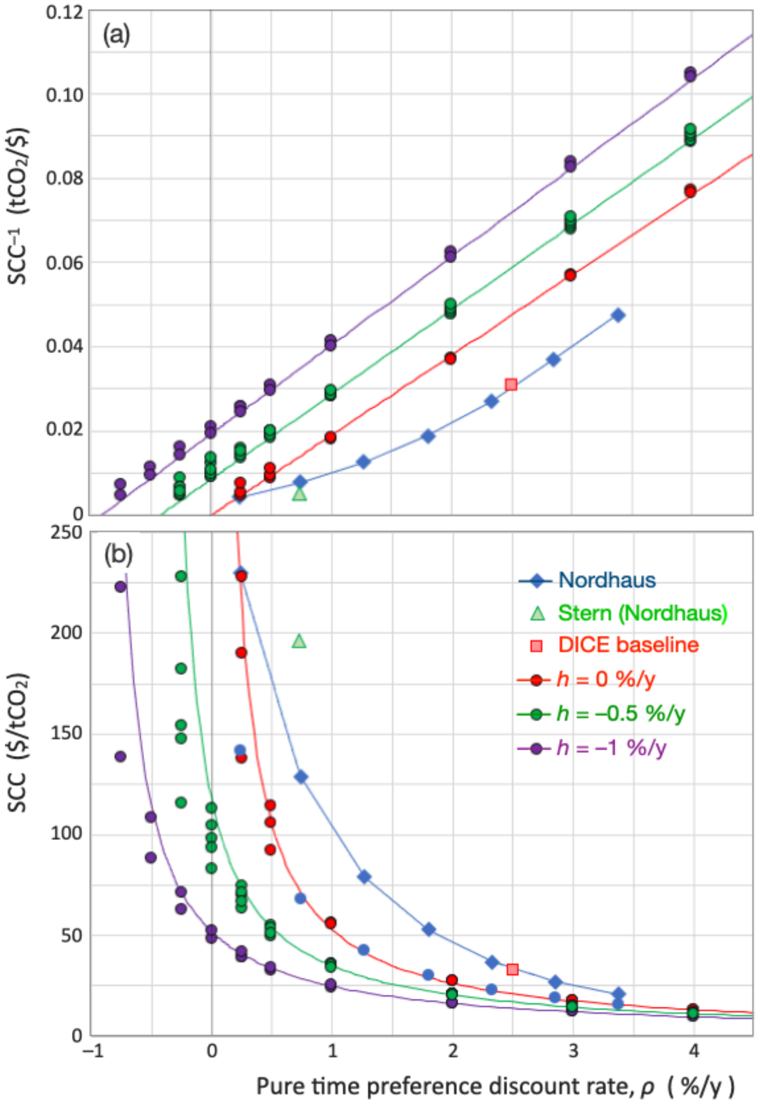

Figure 14. The social cost of carbon SCC (b) and its inverse (a) as a function of the rate of pure time preference ρ for various CO_2 natural growth rates h and various CO_2 emission growth rates C. The Nordhaus, Stern and DICE data are taken from Nordhaus (2017). The lines through are data points in (a) are linear fits for the data points with $C = 0\%/y$. The inverses of the linear fits in (a) are the hyperboles in (b). Note that the blue Nordhaus data in (a) deviate from a straight line, thus from a hyperbole in (b).

The $C = 0\%/y$ data points in (a) lie nicely on straight lines, implying that their inverse –plotted in (b)– must lie on hyperboles. Indeed, Nordhaus claims that the data points should form a hyperbole with an offset of h. However, his blue data points in (a) do not lie on a straight line; in fact, it is a broken line with two slopes and two intercepts. His higher slope is equal to the slopes in my data points; the extrapolated intercept for the low data points is indeed close to –0.5%/y, as remarked by Nordhaus.

Thus, for the simple case where the carbon emission is constant ($C = 0$%/y), the SCC can be easily predicted from the chosen rate of pure time preference ρ plus the natural CO_2 growth rate h. Surprisingly, the SCC goes down both when CO_2 emissions grow ànd when they shrink.

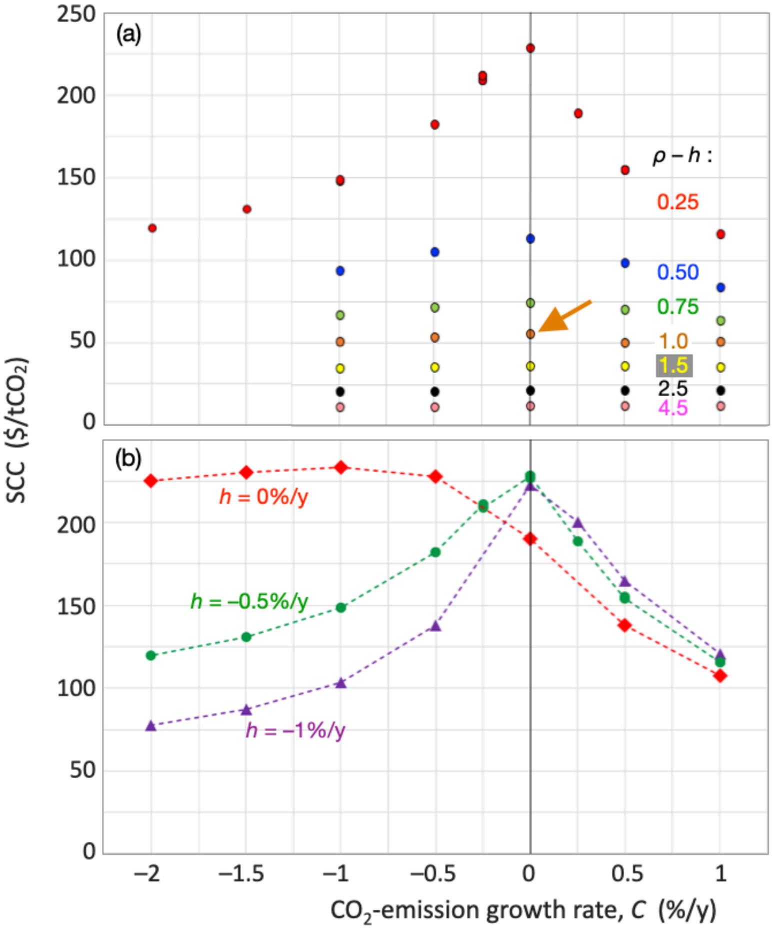

Figure 15. (a) The social cost of carbon (SCC) as function of the CO_2 emission growth rate C for various rates of pure time preference ρ ($h = -0.5$%/y); the arrow points at the central reference data point, to be used in a later section. (b) The same for various *natural* CO_2 growth rates h ($\rho = h + 0.25$ %/y).

Figure 15 shows the SCC as a function of the CO_2-emission growth rate C for various rates of pure time preference ρ. In Fig. 15a, the natural CO_2 growth rate h is –0.5%/y. For ρ only slightly above h (the red and blue data points), the SCC is a strong function of C with a sharp maximum at 0%/y, but the function is asymmetric. Remind that the GDP growth rate g is not important here: as long as the same ρ is used, the same SCC values are obtained for an economy that grows faster or slower. Finally, one sees that for a rate of pure time preference ρ of 1%/y or more above *h*, the SCC in the investigated range varies little, typically by less than 2%. The arrow points to a reference point, to be discussed later.

In Fig. 15b, the SCC for $\rho - h$ = 0.5%/y is shown for *h* = 0%/y, –0.5%/y and –1.0%/y. Surprisingly, for *h* = 0%/y, the SCC is almost independent of the growth rate *C* if *C* < 0%/y. For negative values of the natural carbon growth rates (*i.e. h* < 0%/y), the SCC depends on the carbon growth rate *C.* The drop of the SCC with positive growth rates (*C* > 0) depends only marginally on the natural growth rate *h*.

This partial independence of SCC on *C* if *h* = 0%/y is, however, fortuitous. We know from Eqs. (2) and (3) that $D_r \propto (lnL)^2$; hence: $\frac{dD_r}{dL} \propto 2\frac{\ln L}{L}$ (*L* is expressed here in units of L_p = 278 ppm). The derivative has a broad absolute maximum for *L* = e = 2.718... In particular, for 1.9 < *L* < 4.5, $\frac{dD_r}{dL}$ deviates less than 10% from its maximum. In common units, for the important CO_2 range between 510 ppm and 1280 ppm, the marginal costs for CO_2 emission is incidentally within 10% constant if there is no natural CO_2 decay (thus, *h* = 0%/y).

4.5 Power-law discounting

In my model one can easily replace one discount function by another, for instance the exponential discount function by a power-law discount function. For *w* = 1 in Eq. 7a, one obtains the normal hyperbolic discount function (Rubinstein, 2003).

To test whether this approach leads useful results and new insights in the valuation of future cost by current CO_2 emissions, I did a number of model calculations with the alternative discount function. Table 5 summarizes the input parameters and the SCC results for both the exponential and the power-law discount functions. In all cases, the discount rate $s = \rho + \eta.g$ is 1.9%/y, corresponding to the discount rate found by Giglio et al. (2015).

Two values for the SCC are given: SCC_{1000} is obtained by integration over the entire period of 1000 years, whereas SCC_{∞} is equal to SCC_{1000} plus the estimated contribution of the remaining time, assuming a final constant exponential decay. The latter contribution should be a minor correction, but for the power-law discount function it can become very large and, hence, unreliable. Red numbers in the table refer to unreliable results.

Table 5. The Social Cost of Carbon (SCC), calculated for various GDP growth rates g and carbonization rates C. Two discount function are considered: exponential with discount rate s = 1.9%/y and power-law with w = 2 and t_0 = 56 y. SCC$_{1000}$ is obtained by integration over 1000 years; SCC$_\infty$ includes also an estimate for the contribution until infinity; red values are unreliable. Other parameters: h = –0.5%/y, s = 1.9%/y, η = 1.

	expon.	power	expon.1)	power1)	expon.1)	power1)	expon.	power	unit
g	0		0.5		1		1.5		%/y
SCC_{1000}									
C = –0.5	21.6	18.6	27.7	27.2	38.2	77.6	60.7	1677	\$ / tCO$_2$
C = 0 2)	21.9	18.9	28.1	28.6	39.1 1)	117	62.7	4615	\$ / tCO$_2$
C = 0.5	22.1	18.9	28.3	26.6	38.9	58.0	60.4	716	\$ / tCO$_2$
SCC_∞									
C = –0.5	21.6	18.6	27.7 1)	27.3	38.2	203	60.7	-45	\$ / tCO$_2$
C = 0 2)	21.9	18.9	28.1	29.4	39.1	40.0	62.7	254	\$ / tCO$_2$
C = 0.5	22.1	18.9	28.3	26.7	38.9	77.5	60.4	58.6	\$ / tCO$_2$

1) For the blue cells, see the corresponding curves in Fig. 16.
2) Unit for C is %/y.

One sees that for low GDP growth rates (0%/y and 0.5%/y), the results for the SCC according to the power-law discount function and the exponential discount function are in agreement, the power-law result being slightly smaller, especially at g = 0%/y. Although it is reassuring, the reasonable agreement is not surprising. The discount parameters –*i.e.*, the discount rates s for the exponential function and the power w and time factor (t_0)– are chosen such that they describe the data in the underlying study by Giglio et al. at least in the important 80-250 y range. In fact, the lower results for the power-law function at g = 0%/y is related to the effective higher discount rate for short (<50 y) maturities. Figure 16 shows why. Here, the power-law function has a steeper slope near t = 0 than the exponential function, meaning a temporary higher initial effective discount rate. The effect that the power-law discounting yields a lower SCC disappears if the economy grows (g > 0%/y) and marginalizes the contributions to the SCC from the early years.

Figure 16 illustrates the calculation of the time-differential SCC$_p$(t) and SCC$_e$(t) for various input parameter values. In (b) and (c), there is no net increase nor decrease of CO$_2$ and the natural CO$_2$ growth rate h of –0.5% leads to saturation of the CO$_2$ level close to 1000 ppm and a temperature rise of 5 to 6 °C (light green curve). Consequently, the relative economic damage saturates at 7.0% (not shown) and the absolute damage function (brown curve) runs parallel to the GDP curve, shifted downward by a factor of 14. The discounted damage function is shown in red discount function). The important curves are the dark green ones: the time-differential SCC$_p$(t) (in full) and SCC$_e$(t) (dashed). Their integrals are the SCC$_p$ and SCC$_e$, respectively, see eq. (8d).

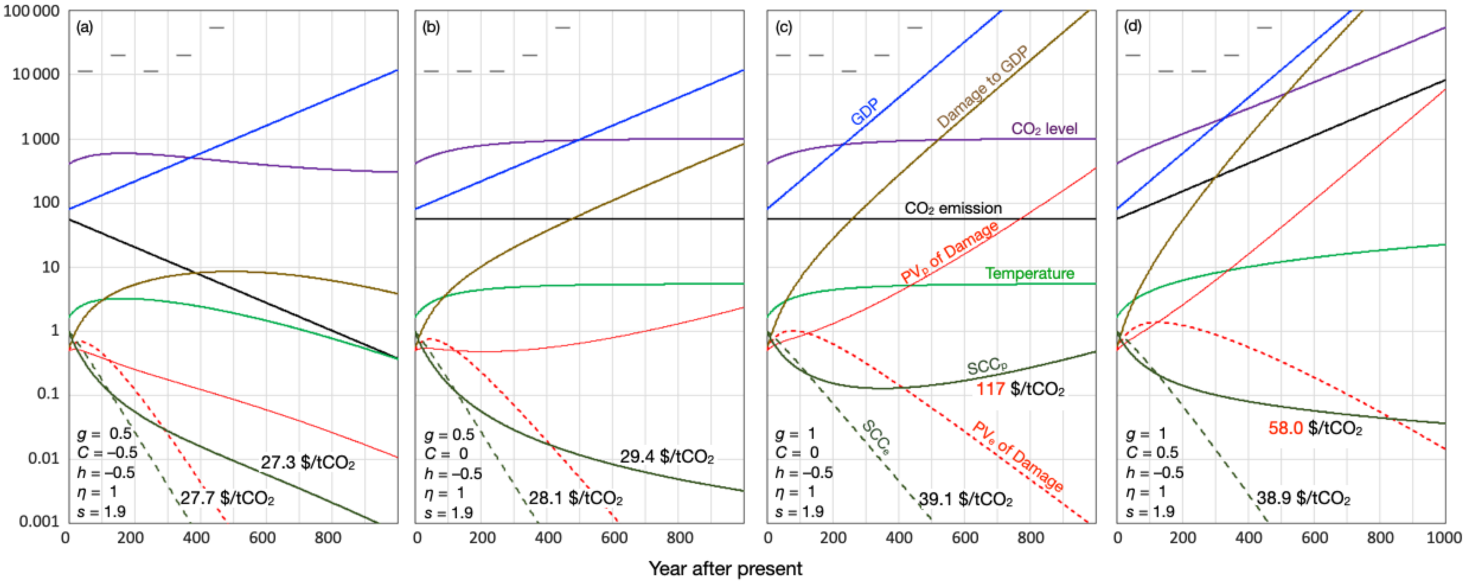

Figure 16. Time-evolution of a number of economic parameters. The varied parameters are *g* and *C*. In each panel, the final SCC for exponential (SCC_e) and power-law discounting (SCC_p) are mentioned, red values are not reliable. See also table 4.

In (c) and (d) the growth rate *g* is relatively high –see the steep GDP line– and, hence, the power-law $SCC_p(t)$ bends up eventually, making the outcome of $SCC_{1000,p}$ undefined. Also, the correction cannot be applied. For the moderate *g* in Fig. 16a+b, $SCC_p(t)$ deceases with time at least until year 1000. Note that if there is even a minor positive GDP growth rate, the $SCC_p(t)$ curve will eventually bend upward and its integral becomes infinite. Therefore, I conclude that in this form the method does not lead to useful outcomes for many realistic situations. The solution is to add constraints (*e.g.*, a finite integration period) are applied. However, proper application of new constraints is not trivial and can only be applied safely after sufficient mathematical analysis; it is not pursued further here.

Despite some uncertain results, the outcomes of the power-law discounting do provide insights into the influence of the method of discounting on the calculated social cost of carbon. For more, see the Discussion and Conclusion section.

4.6 The effect of a temperature limit / CO_2 budget

Decarbonization of the economy ($c < 0\%/y$) reduces CO_2 emission and is an obvious measure to limit climate change. In a previous section I have shown that the carbonization rate *c* has an effect on the SCC. An extreme measure of decarbonization is to stop CO_2 emission 'overnight', for instance because it becomes suddenly clear that a certain tipping-point temperature threatens to be reached.

For the case of $g = 1\%$/y and $C = 0.5\%$/y, I did additional calculations with strict temperature limits. If a temperature limit is reached, all human CO_2 emissions terminate immediately and forever. Figure 17 shows the corresponding evolution of key variables.

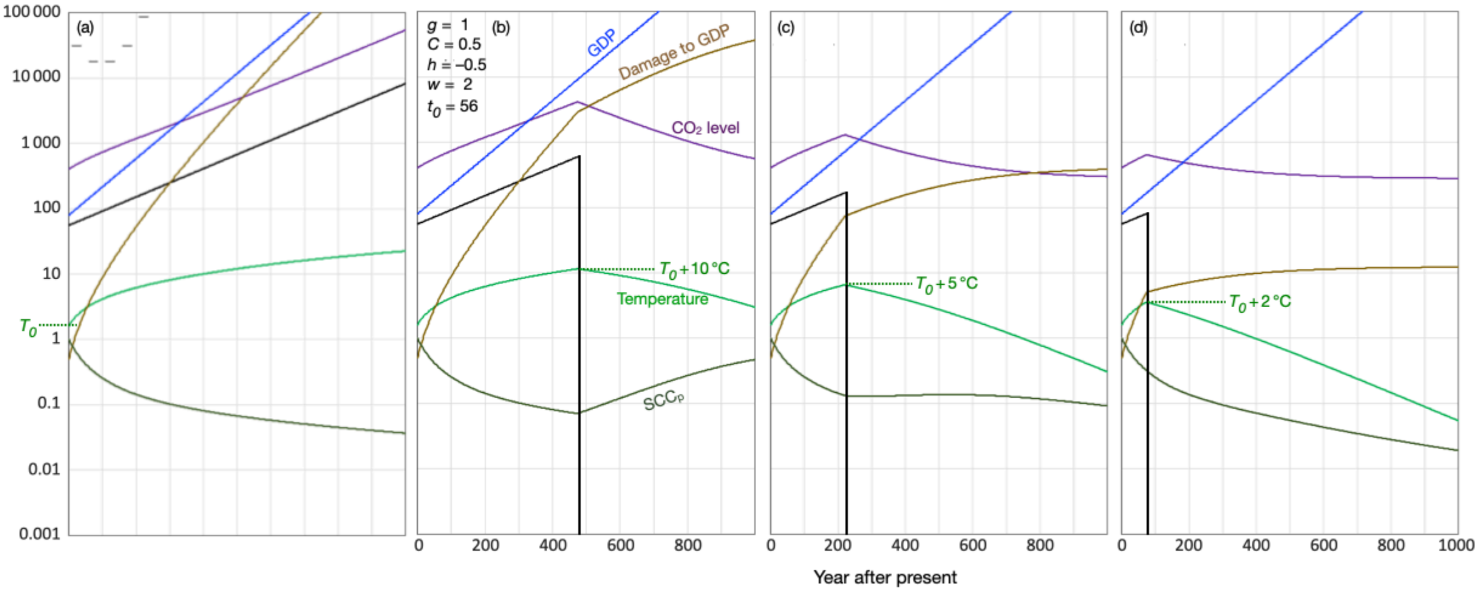

Figure 17. The evolution of variables if emissions terminate when a certain temperature limit is reached: (a): no limit, related to Fig. 16b. (b): 10 °C, (b): 5 °C, (c): 2 °C above the temperature in year 0. (Power-law discount function.)

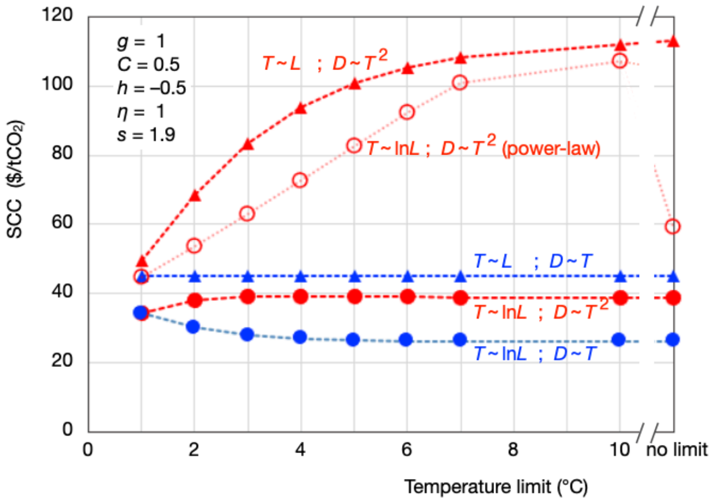

Figure 18. SCC as function of the temperature limit for various climate sensitivity and damage models. For the default situation, T~ln$L$ (circles): the temperature increases proportionally to the logarithm of the CO_2 level L. For T~L (triangles), the temperature increases proportionally to the increase in CO_2 level. D~T^2 (red data) indicates that the damage is proportional to the temperature increase squared (default); D~T (blue data) indicates that the damage is proportional to the temperature increase. The open red circles refer to the power-law discounting (default conditions).

One sees that the long-term tail of the $\mathrm{SCC_p}(t)$ curve contributes in all cases substantially to the integral $\mathrm{SCC_p}$. In fact, for a limit of 10 °C, the $\mathrm{SCC_p}$ is even higher than without the limit. Figure 18 plots the $\mathrm{SCC_p}$ and $\mathrm{SCC_e}$ as function of the temperature limit.

Surprisingly, the SCC depends in most situations weakly on the temperature limit for the exponential discount function. For the power-law function, the result is heavily influenced by the tail and is likely an artefact of the large GDP growth and the rather weak long-term discount. The default conditions in this work are: T proportional to $\ln L$ and damage proportional to T^2, see the filled red circles. If one takes T proportional to L (upper red triangles), then the SCC decreases substantially with decreasing temperature limit (or CO_2 budget). This result is not surprising, because in the linear model, the temperature rises much stronger than in the logarithmic model. If these higher temperatures are banned, the gain will be high. In contrast, if one assumes the damage to be proportional to the temperature (blue circles), then the higher temperatures are much less pernicious; hence, its avoidance is less profitable. In fact, the SCC increases a bit when the temperature limit is reduced down to 1 °C.

The discrepancy between the two discount functions with default conditions (filled and open red circles) disappears with decreasing temperature limit. Obviously, the effects of the tail in Fig. 17 weaken when the high CO_2 levels in later centuries with high GDP values are avoided.

4.7 Effect of the risk-avoidance or equality parameter η

In Ramsey's formula (Eq. 1) the parameter η expresses risk avoidance or preference for equality. Both Nordhaus and Stern use for simplicity $\eta = 1$ and so far in this work I do the same. Dasgupta (2007) states that a value for η of 1.5 or higher should reflect better the call for more equality between richer and poorer countries.

Table 6. The change in SCC if η is changed from 1 to 1.5 for various GDP growth (g) and carbonization rates (c) and for two pure rates of time preference, $\rho - h$ ($h = -0.5\%/\mathrm{y}$). Variables *c, g* and *s* are expressed in %/y. The value of 0.71 at the top of the 2nd data column *e.g.* signifies that SCC($\eta = 1.5$) is 71% of SCC($\eta = 1$), other parameters being equal.

	$\rho - h$ = 0.5				$\rho - h$ = 2.5			
	$g = 0$	$g = 0.5$	$g = 1$	$g = 1.5$	$g = 0$	$g = 0.5$	$g = 1$	$g = 1.5$
$c = 0$	1.00	0.71	0.61	0.55	1.00	0.90	0.83	0.77
$c = -0.5$	1.00	0.66	0.55	0.50	1.00	0.90	0.83	0.76
$c = -1$	1.00	0.68	0.51	0.44	1.00	0.90	0.82	0.76
$c = -1.5$	1.00	0.71	0.49	0.39	1.00	0.90	0.83	0.76
Eq. 9	1.00	0.67	0.50	0.40	1.00	0.91	0.83	0.77

Table 6 presents the outcomes of calculations with $\eta = 1.5$. In these calculations, $h = -0.5\%/y$ and ρ is either 0%/y or 2%/y. In absolute numbers: the reduction SCC varies between 27 and 69 \$/tCO_2 for $\rho - h$ is 0.5%/y and between 2 and 5 \$/tCO_2 for $\rho - h$ is 2.5%/y. Not surprisingly, the effect of the inclusion of risk avoidance or enhanced equality is mainly a function of the GDP growth rate; it depends only marginally on the decarbonization rate. In all cases, the effect is a reduction. For $g = 0\%/y$, there is of course no effect of a different choice for η.

In the Ramsey formula of Eq. (1a), a change in the $(\eta - 1).g$ term has the same effect as a change in the ρ term. Therefore, the $(\rho - h)^{-1}$ relation could be extended to $(\rho + (\eta - 1).g - h)^{-1}$. Indeed, the last row gives 'theoretical' numbers in reasonable agreement with the other table elements, calculated via the generalized formula (Eq. 9) in the Discussion and Conclusion chapter.

4.8 Comparison of (combined) power-law discount with Nordhaus 2017 data

The question arises naturally what the SCC will be if the power-law discount function is being applied to a more realistic prediction. Obviously, at present we have only predictions or scenarios. As a first attempt, I use the DICE base data of Nordhaus (2017). Nordhaus assumes a decarbonization rate of 1.5%/y and mentions a discount rate for the DICE baseline scenario of $s = 4.25\%/y$ and growth rates g of the GDP per capita dropping from 2.1%/y in the first half to 1.9%/y in the second half of the 21st century. Unfortunately, his paper is not fully internally consistent nor clear. Its Fig. 3 depicts the DICE baseline at $s - g = 2.55\%/y$, implying $g = 1.7\%/y$, inconsistent with the abovementioned values of 2.1 to 1.9%/y. Furthermore, he does not specify the parameter δ (in this work $-h$, minus the natural growth rate of CO_2), but the value used could be 0.5%/y, as in a previous publication (Nordhaus, 2014). Moreover, the extrapolated intercept of his data in Fig. 14a is consistent with $\delta = 0.5\%/y$.

For my model calculation I use the parameters in the last data column of Table 1. I assume that the growth rate g starts at 2.1%/y in 2010, dropping linearly to 1.3%/y in 2100, thus with an average of 1.7%/y in these 90 years. After 2100 it continues to drop and reaches 0%/y in year 2244, to remain at 0%/y henceforth. For $\rho = 2.55\ \%/y$ and $h = 0\%/y$, the SCC is 21.9 \$/tCO_2, see Table 7.

Table 7. Comparison between exponential and power-law discounting for the DICE baseline model, with *g* varying in time.

	exponential time preference		power-law time preference		unit
h	0	0	0	0	%/y
ρ	2.55	1.41	-	-	%/y
β	-	-	0.66	-	%/y
w	-	-	1	2	
t_0	-	-	38	56	y
$\mathrm{SCC}_{e/p}$	21.9	41.0	41.0	134	$\$/\mathrm{tCO_2}$
h	–0.5	–0.5	–0.5	–0.5	%/y
ρ	2.55	1.41	-	-	%/y
β	-	-	0.66	-	%/y
w	-	-	1	2	
t_0	-	-	38	56	y
$\mathrm{SCC}_{e/p}$	17.4	29.0	29.0	63.5	$\$/\mathrm{tCO_2}$

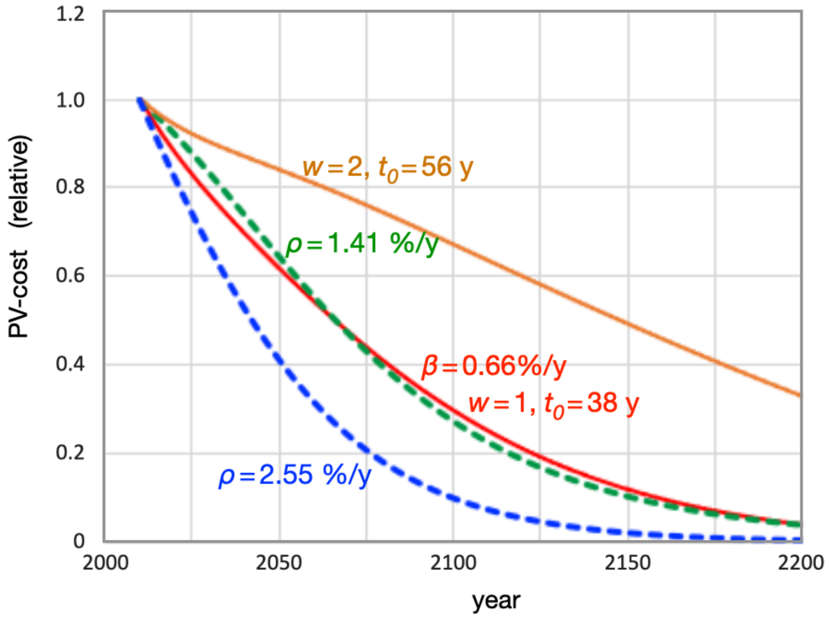

Figure 19. Discounted value of the costs of climate change damage due to CO_2 emission in 2010 for the DICE baseline climate model (Nordhaus, 2017). The integral of a curve equals SCC, see Table 7. Dashed curves: exponential discounting; full curves: power-law discounting.

The calculation of the SCC for the DICE baseline yields only two thirds of the DICE baseline of 31.2 $\$/\mathrm{tCO_2}$ in Nordhaus (2017). I have no explanation for this difference. If the power-law discount function for the pure time preference is used according to the study of Giglio et al. ($w = 1$, $t_0 = 38$ y, $\beta = 0.66$%/y) plus a constant GDP per capita growth rate of 1.7%/y, then the SCC is 41.0 $\$/\mathrm{tCO_2}$. The same SCC can be obtained if one applies a rate of pure time preference ρ in exponential discounting of 1.41%/y, see the almost overlapping red and green curves in Fig. 19. If a $w = 2$ power-law is used, the PV decays much slower and the SCC is about twice as large, see Table 6.

These results show that in the DICE baseline scenario the SCC is about twice as high if the long-term discount observations of Giglio et al. (2015) are used. The use of these observations could be regarded as a drop of the applied rate of pure time preference from 2.55 %/y to 1.41%/y.

Still the question remains open whether or not power-law discounting provides better calculational results or gives new insight in the Stern-Nordhaus debate. Figure 19 shows that that power-law discounting couples in a smooth manner short time (<25 y) 'high' market interest rates to long-run low interest rates. For $t = 0$, the absolute value of its (relative) derivative is $\beta + 1/t_0$ = 5.6%/y. For 100 y, it is 1.4%/y and the final value is 0.6%/y. In this way, both viewpoints are incorporated in power-law discounting: short term market rates and long term psychological-ethical acceptable rates.

5. Discussion and Conclusion

Discounting in financial transactions is common practice and undisputed. However, in project planning over long periods it poses problems, because the basic parameter in discounting –the discount rate– is usually derived from the cost of capital (Palepu et al., 2019), which is linked to financial markets. The actual interest rate for instance can change over time scales much shorter than the usual life time of large projects. The rentability of personal pension plans *e.g.* are affected by interest rate fluctuations, putting stress on this saving system. In general, future costs and profits and current investments that influence those future costs or profits cannot be weighted well against each without proper discounting. This holds also for investments to abate future costs of climate change caused by current greenhouse gas emission, in particular CO_2 emissions. In fact, discounting is a key element in integrated assessment models for calculating the cost of climate change abatement, see a recent example by Van der Wijst, Hof, and Van Vuuren (2021).

A core issue in the controversy between Stern and Nordhaus on the costs of climate change is the application of the discount factor over decades or even centuries. Stern proposes 0.1%/y, which is in accordance with the Jonas *Imperative of Responsibility* (1997) to attribute equal worth of all human lives, independent of time. Nordhaus on the other hand claims that in all financial projects the market discount rate must be used, also in climate change abatement. However, this controversy has a societal, even an ethical base and hence cannot be answered in financial terms solely. From real-world observations Giglio et al. (2015) conclude that a market-compatible discount rate cannot describe the long-run discount that households apply for residential property. My analysis of Giglio's data shows that households use a long run discount factor that can be described both by a power-law function of power of 2, alternatively by a power-law function of power 1 multiplied by an exponential function with a slow constant decay rate (~0.5%/y), the latter also proposed by recently Giglio.[27] In order to decide which function is best, more observations are needed, especially for valuations over 250 years or more. Additional analysis shows that the exponential function with a decreasing discount rate recommended by the HM Treasury (2003) can also be described as a power-law discount.

One can derive support for power-law discounting from behavioral economics and from psychology. This support is, however, not strong, since the use of lab studies or field observations to uncover people's attitudes to future and to transgenerational costs and profits remains questionable. Drawing guide rules for the protection of a healthy biosphere from

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[27] https://www.youtube.com/watch?v=7CpTAaSl6jI (same as note 26).

people's desirability to receive money on a certain date or even from the price of multi-century leasehold residential properties has some advantage, but limited.

Clearly, a rate of pure time preference of 1%/y that Giglio et al. (2015) finds is much lower than the 'growth corrected discount rate' of 2.4%/y Nordhaus (2017) uses for the DICE baseline IAM calculation of the social cost of carbon (SCC). However, it is at the same time much higher than the 0.1%/y rate of pure time preference proposed by Stern (2006). One could be tempted to conclude that the truth can be found halfway, but this practical solution to the controversy is too simple. First of all, confronting Giglio's observations to the DICE baseline, I conclude that in the latter a rate of pure time preference of 1.4%/y should be used. This reduction would almost double the SCC. Secondly, financing is an important element in the debate on climate change mitigation and adaption, but the moral or ethical viewpoint should not be subordinate to that, as stated by Ferrari & Mery (2008) and by Neumayer (2007). Indeed, recent jurisdiction has put the well-being of future generations as a concern for current policy makers (Bundesverfassungsgericht, 2021), which could be translated in financial-economic terms as the use of very low (if not zero) rates of pure time preference. Thirdly, the GDP growth related part in the Ramsey formula of the discount rate (g in Eq. 1) could lead to negative discount rates if GDP growth (per capita) becomes negative (Koopmans, 1965), maybe because of planetary boundaries (Raworth, 2017). Although calculating the costs of climate change mitigation and adaptation remains indispensable, I conclude that a pure financial discussion leads us astray.

Instead of a practical 1%/y middle-ground solution to the Stern-Nordhaus controversy, I put forward the use of UK Treasury –or whatever central bank– long-term interest rates as a basis for discounting in IAMs. A similar approach is suggested by Gollier et al. (2008). But instead of interest tables, smooth discount functions can be used, for example Eq. 7b with $w = 1$, $\beta = 0.58\%/y$ and $t_O = 21$ y.

In debates about the best response to climate change, IAMs are essential tools and financial-economic considerations remain crucial elements in IAMs. A major shortcoming of IAMs is, as Dasgupta (2007) notes, the intractibility between input data and assumptions and output. In this study I investigated a number of fundamental dependencies of the SCC, that will help us to discern trends and hidden relationships. Figure 14 shows for instance that the SCC is inversely proportional to the sum of the rate of pure time preference ρ and the natural carbon growth rate h. Taking all dependencies found in the previous chapter together, one can construct an approximate formula for the SCC. If the absolute CO_2 emissions drop ($C < 0\%/y$), the relationship becomes surprisingly simple:

$$SCC(g,h,C,\rho,\eta)=\frac{\mathrm{A}\,\varphi_2 G_0}{\rho+(\eta-1)g-h} \quad . \qquad \text{(Eq. 9a)}$$

Here, A is a proportionality constant, φ_2 is the quadratic prefactor of the damage function in Eq. 3, G_0 is the size of the global economy at time 0. Note that it is assumed that all growth rates, positive or negative, are constant. The $\rho - h$ term in the denominator has also been mentioned, though not explained, by Nordhaus (2017). The arrow in Fig. 15a points to a reference point, where $\mathrm{SCC}(g=1,h=0,C=0,\rho=1,\eta=1)=\frac{\mathrm{A}\,\varphi_2 G_0}{\rho}$ = 55 \$/tCO$_2$. Since φ_2 = 0.236%/°C^2 and G_0 = 80 T\$/y, the proportionality constant A ≈ 2.9 x 10^{-12} °C^2/tCO$_2$.

Although the overall equation 9a is phenomenological and not analyzed in all detail in this work, it is in accordance with the analytical formulation derived by Van den Bijgaart et al. (2016).[28] The usefulness of Eq. 9a and of the formulation by Van den Bijgaart lies in the ability to understand or predict trends in reality or in much more complicated model calculations, *e.g.* when comparing different IAM outcomes. For instance, one sees that the SCC is proportional to the parameter φ_2, that links GDP damage to increased temperatures, though still assuming a quadratic relationship. Nordhaus (2017) takes φ_2 equal to 0.00236 °C^{-2}; Howard & Sterner (2017), in contrast, use a four times higher value. Who is right remains unanswered yet, but with the simple proportionality one can focus discussions more easily onto the essentials. Of course, also the assumed dependence on the temperature *squared* is debatable. Burke et al. (2015) find a much more complicated relationship between global or regional temperature changes and economic damages. Moreover, Eq. 9a cannot describe effects of tipping point temperature.

If C > 0%/y, a situation that is not sustainable, the SCC needs a correction. For 0%/y < C < 1%/y, it becomes:

$$\mathrm{SCC}(g,h,C,\rho,\eta)=\frac{\mathrm{A}\ \varphi_2 G_0}{\rho+(\eta-1)g-h}\,(1-\mathrm{B}C) \quad , \qquad \text{(Eq. 9b)}$$

where constant B is estimated from Fig. 15b, yielding B ≈ 50 y. For the unrealistic C > 1%/y, I have not checked the relationship.

Equation 9 attests also how different disciplines contribute in the climate change debate. The relevance of their contributions, as far as discussed in this paper, are summarized in the table below. For instance, we can draw conclusion about the natural growth/decay rate h of excess CO$_2$ in the atmosphere. It is decaying when h < 0%/y, thus when CO$_2$ is taken up by buffers (*e.g.*, oceans or vegetation). However, buffers could in a later phase release captured CO$_2$, causing

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[28] In particular, Eqs. 23 & 25/26 in Van den Bijgaart et al. (2016), with the assumptions and parameters of the present work: l = 0%/y, ψ = 2, ξ = 1, and $\varepsilon \gg \delta$.

intricate feedback loops and reversal of trends. In that case, *h* increases and might become positive. Eq. 9 shows that an increasing value of *h* will increase the SCC.

Table 8. The role of various disciplines in the discussions of costs of climate change abatement, as discussed in the present study.

	natural CO_2 exchange	*damage function*	*GDP and its growth*	change in carbon use	rate of pure time preference	equality
discipline	h	φ_2	$G_0 ; g$	C	ρ	η
climatology, geosciences	+	+				
technology		+	+	+		
economics		+	+	+	+	+
finance			+		+	+
ethics and law					+	+
psychology &sociology					+	+

The model developed and used in this study does not include any economic optimization. Nevertheless, together with real-world data it can sift in debates non-crucial and fortuitous differences or resemblance from the essential ones. In this way, the first research question of section 2.5 can be answered positively: it does indeed provide new insights. Also different functions than exponentials can be used for discounting, although –apart from financial, psychological or ethical issues– calculational complications can arise.

The proposed simplified and abstract formula put forward here (Eq. 9) can indeed approximate the social cost of carbon. Although the approximation has limitations –not all of them studied here– the answer to the second research question is also positive.

Despite the new insights into fundamental parameters that underlie the social cost of carbon, it is clear that the SCC is by far not the most important parameter. Apart from the unquantifiable and invaluable worth of a rich and resilient biosphere, the preservation of the biosphere is a concern that derives its importance principally from moral and ethical beliefs (Ferrari & Mery, 2006; Neumayer, 2007). Natural sciences, economics, sociology, jurisdiction, and technology contribute, see Table 8. In practice, best policy would be to investigate the consequences of human's present and possible future activities and to reflect on the choices that can or must be made. Anyhow, complex IAMs remain crucial to understand the consequences of these activities and to assess on a broad stage as possible the consequences of choices and viewpoints.

Finally, I note that discounting should not dominate societal debates, if only because time itself is a property of patterns of human information processing (Michon, 1972). Probably Jorge Luis Borges describes time best (p. 1).

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Appendix 1

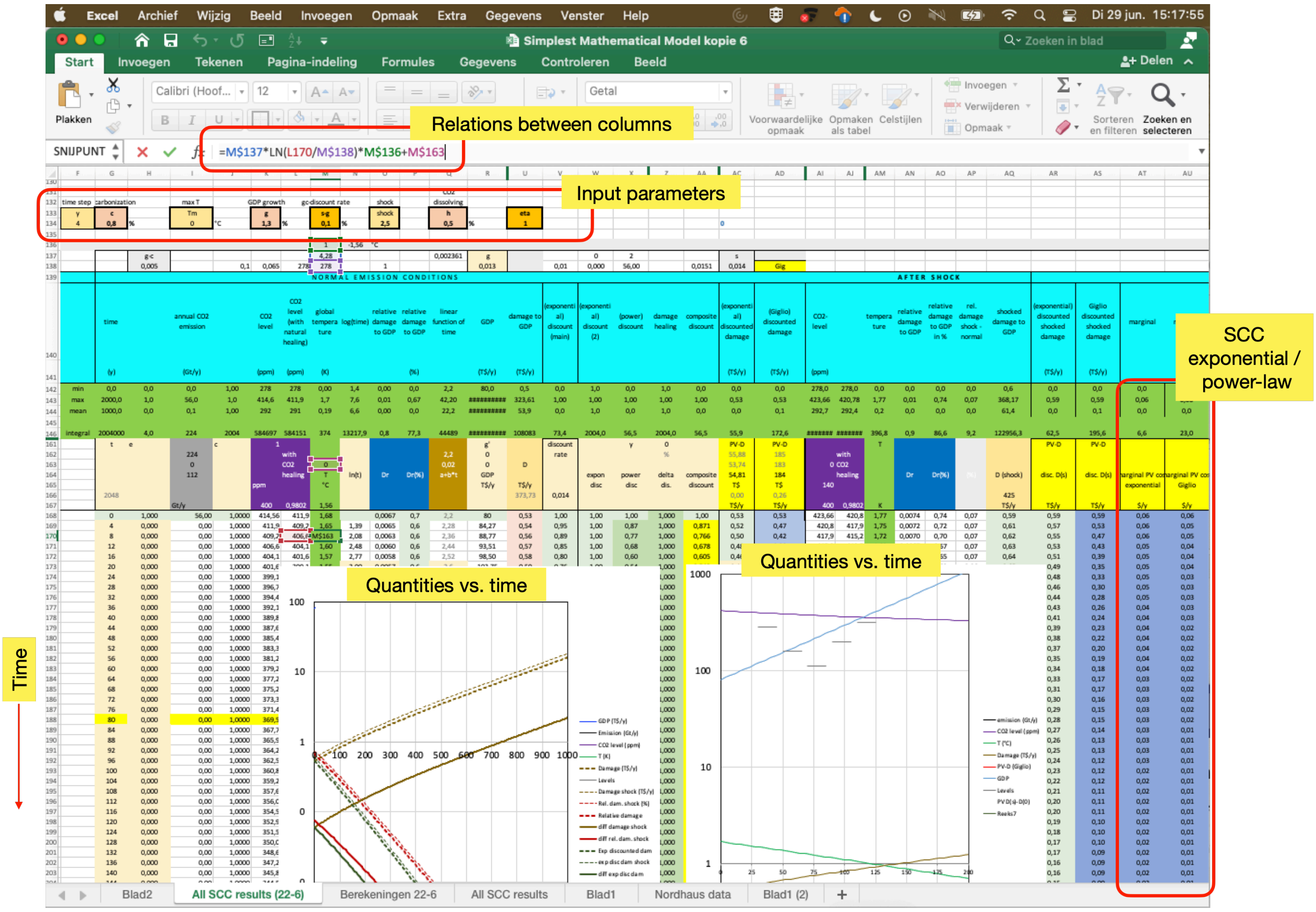
Overview of all parameters, variables and functions used:

Variable	remark or alternative name	symbol	lower bound 1)	upper bound 1)	Nordhaus 2017 7)	unit
present			2020		2010	y
year	year 0 = is present	t	0	1000	0...1000 8)	year or y
time step	in the calculations	dt	2	10	2	year or y
GDP growth rate	GDP per capita growth rate	g	0	1.5	2.1...0.0 8)	%/y
carbonization rate	change in global CO_2 emission / GDP	c	–1.5	0	–1.5	%/y
C	growth rate in carbon use ($= g + c$)	C	–1	1	var.	%/y
natural CO_2 growth rate	($-\delta$ or $-\delta_S$ in the literature)	h	–1	0	–0.5 / 0 9)	%/y
discount rate	(if exponential)	s	$g + h$	$g + h + 4$	4.25	%/y
rate of pure time preference	growth-corrected discount rate in Nordhaus	ρ ($= s - g$)	$-h$	4	1.42 / 2.55 9)	%/y
equality parameter	also called risk-avoidance parameter	η	1	1.5	1	
emission	start CO_2 emission	E_0	56	4)	33	Gt/y
emission	CO_2 emission in year t	$E(t)$	6)	6)	6)	Gt/y
shock emission	extra emission pulse (in year 0)	Γ	1 x E_0 2)	100 x E_0 3)	1	Gt
size of economy	start size global economy, GDP	G_0	80	4)	66	T$/y
size of economy	GDP, function of time t	$G(t)$	6)	6)	6)	T$/y
CO_2 level	in the atmosphere	$L(t)$	6)	6)	6)	ppm
	pre-industrial atmospheric CO_2 level	L_p	278	4)	278	ppm
	start atmospheric CO_2 level	L_0	400	4)	380	ppm
elevated temperature	start global temperature above pre-industrial value	T_0	1.56 5)	4)	1.35 5)	°C
elevated temperature	global temperature above pre-industrial value	$T(t)$	6)	6)	6)	°C
climate sensitivity	equilibrium temperature at double L_p	S	2.95	4)	2.95	°C
	final temperature (in year 1000)	T_f	6)	6)	6)	°C
maximum temperature	emergency stop temperature for CO_2 emissions (= CO_2 budget)	T_x	2	unbounded	-	°C

	maximum temperature reached in investigated time range	T_m	6)	6)	6)	°C
	year of maximum temperature	y_m	6)	6)	6)	y
	linear damage coefficient	φ_1	0	4)	4)	% . °C^{-1}
	quadratic damage coefficient	φ_2	0.236	4)	0.236	% . °C^{-2}
relative damage	relative to GDP	$D_r(t)$	6)	6)	6)	%
	absolute damage: $D_r(t)$ x $G(t)$	$D(t)$	6)	6)	6)	T$/y
discount factor	for exponential discounting	$f_e(t)$	1	0	6)	
discount factor	for lower-law discounting	$f_p(t)$	1	0	6)	
present value		PV				
	cost	$C(t)$	6)	6)	6)	T$/y

Appendix 2

The model is set up in excel. The figure below is an example of some of the calculations.



=M$137*LN(L170/M$138)*M$136+M$163