Effect of correlated uncertainties in economic growth and CO_2 emissions on the social cost of carbon

Paul Alkemade

Delft University of Technology, Delft, the Netherlands

Climate change generate uncertainties. For effective countermeasures, one must predict possible outcomes of policy actions. To describe potential scenarios, one uses Integrated Assessment Models (IAMs), but because of fundamental uncertainties, the likelihood of specific scenarios is difficult to assess, even if their probability distributions are known [Barnett *et al.*, 2022]. Only a few IAM studies incorporate risks (*i.e.,* known uncertainties), sometimes correlated risks, either by Monte Carlo methods [*e.g.* Rennert *et al.*, 2022a+b] or by simple analytic formulas [*e.g.* Van den Bremer and Van der Ploeg, 2021].

Here, I use the moments of probability distributions to study correlated risks in an analytic IAM. This work shows quantitatively that the social cost of carbon (SCC) depends on both the risks and their correlations. The current positive correlation between economic growth and CO_2 emission decreases the SCC, but a transition to green growth might reduce this correlation, thus increasing the SCC.

This work uses a versatile analytic integrated assessment model, *Matrice* [Alkemade, 2023]. The main difference with simpler analytic IAMs lies in the inclusion via Laplace transforms of many terms that describe involved processes. Using derivatives and the moments of the underlying probability distributions, one can approximate the expectation value of stochastic outcomes. The expectation value E[S_{CC}] of the SCC that depends on two stochastic variables is

$$\mathrm{E}[S_{cc}(x_1, x_2)] = \sum_{i,j} \frac{1}{i!\,j!} M_{ij}^{x_1^*, x_2^*} \left(\frac{\partial^{i+j} S_{cc}}{\partial x_1^i \partial x_2^j} \right)_{x_1, x_2 = x_1^*, x_2^*} .$$

The moments of the two-dimensional probability distribution $P(x_1, x_2)$ are $M_{ij}^{x_1^*, x_2^*} = \iint (x_1 - x_1^*)^i \, (x_2 - x_2^*)^j \, P(x_1, x_2) \, dx_1 \, dx_2$. If x_1^*, x_2^* are the expectation values $\overline{x_1}, \overline{x_2}$, then M_{20} and M_{02} are their variances and M_{11} is linked to their correlation coefficient. Note that one can likewise calculate the variance and skewness of the SCC.

The influence of probability distributions for economic growth and for CO_2 emission, including correlations, on the social cost of carbon (SCC) is the subject of this study. The study follows the work by Rennert *et al.* [2022a+b]. In particular, I convert their quantiles for GDP/cap growth and emissions into full distributions. I use the same relation between CO_2 level and global temperature and take the same elasticities of marginal utility η and pure-time preference discount rates ρ. For constructing skewed probability distributions, I apply the method by Van den Bremer and Van der Ploeg [2021]. From two one-dimensional distributions I construct single two-dimensional distributions with correlation coefficients between –0.75 and +0.75. Next, moments up to M_{44} are calculated. Further, I assume a global economy without regions or sectors. The population is from [Rennert *et al.*, 2022a+b], but without an uncertainty distribution.

I take three distinct damage functions: DICE [Nordhaus 2017], Howard *et al.* [2017] and Burke *et al.* [2015]. The DICE model assumes a quadratic relation between global temperature change and economic productivity loss. For the compilation by Howard *et al.* [2017] I use third-order fitting [Alkemade, 2023]. Finally, the Burke function is fitted by a fourth-order function in temperature, though its major component is a linear term [Alkemade, 2023]. Quantitatively, these damage functions differ substantially; however, this work focuses on *relative* effects. Derivatives up to 4th order are approximated by calculating SCC at a 5x5 grid for GDP growth and CO_2 emission around their respective expectation values.

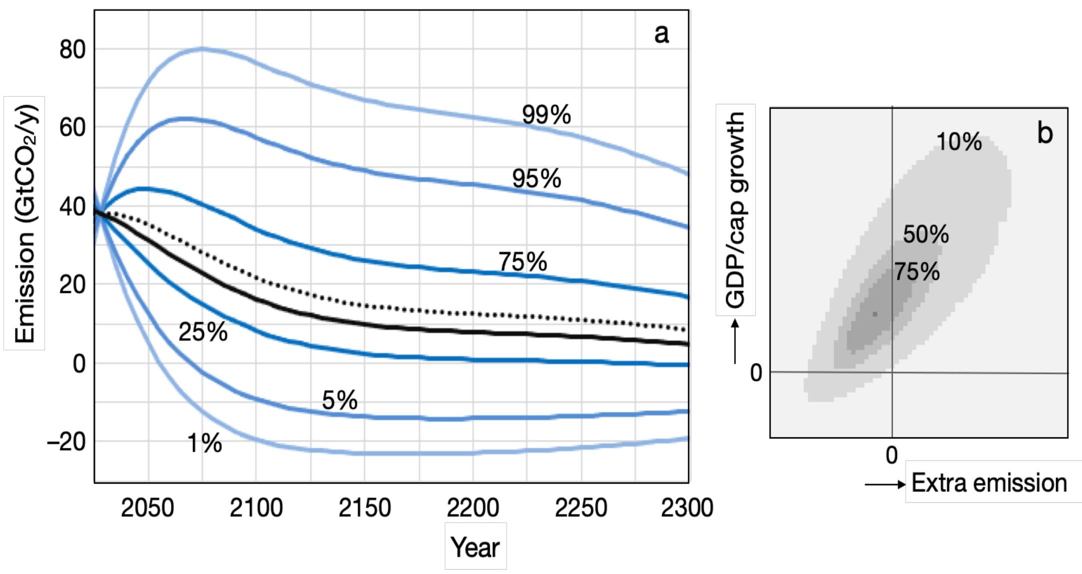

Figure 1. a) CO_2-emission scenarios. The quantiles are the probabilities that the actual emission scenario lies below the corresponding curve. The black curve is the median, the dotted one the expectation curve. (Data derived from [Rennert *et al.*, 2022a+b]). b) Map of a two-dimensional skewed probability distribution with +0.75 correlation.

Figure 1a shows quantiles of CO_2 emission scenarios. As Rennert *et al.* [2022a+b], I assume that the expected global GDP/cap grows first by 1.5% per year until 2100, then declines to 1%/y in 2200, remaining there until 2300. Figure 1b is an example of a correlated and skewed probability distribution.

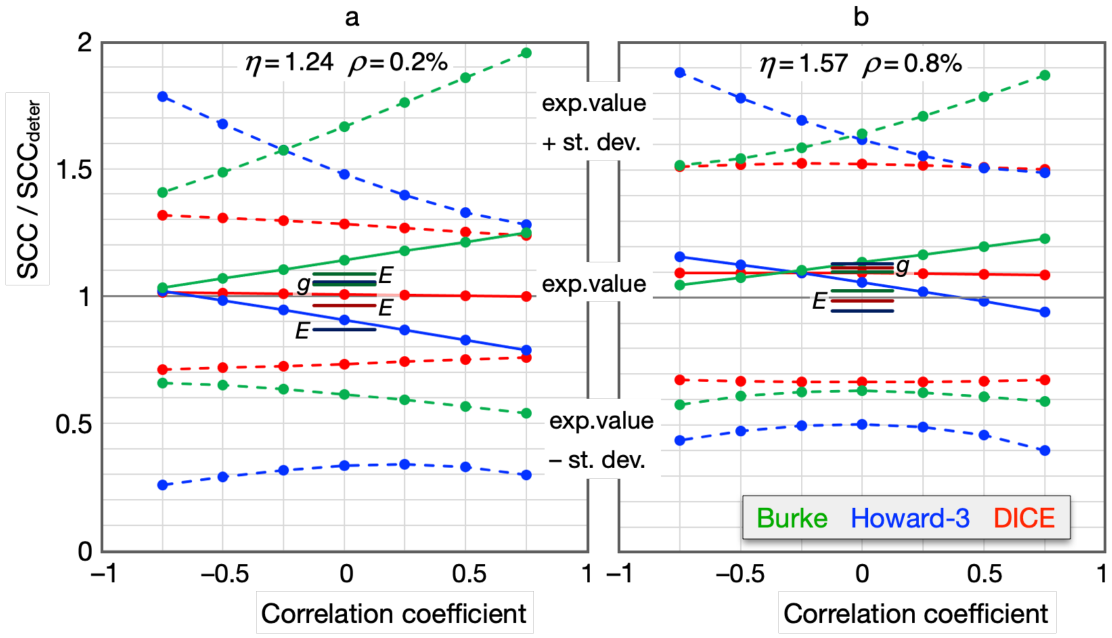

Figure 2. The relative effect of uncertainties and correlations in the GDP/cap growth rate and CO_2 emissions on the SCC (SCC$_{deter}$ is the deterministic SCC, *i.e.* without uncertainties). Full curves depict expectation values; dashed ones are expectation values plus or minus the standard deviation. (The area between the dashed curves covers about 2/3rd of all outcomes.) The dark stripes at the centers indicate the outcomes when only uncertainties in economic growth ('*g*') or in emissions are ('*E*') considered.

Figure 2 shows for two η-ρ combinations SCC/SCC$_{deter}$ versus the correlation coefficient. The absolute SCC in 2a is 106, 438 and 1907 \$/tCO_2 for the DICE, Howard-3 and Burke damage functions, respectively.[1] For 2b, SCCs are twice as low.

One sees that the effect of uncertainties depends on the convexity of the damage function. For a quadratic function (DICE, red) the effect in Fig. 2a is virtually zero. For the third-order damage function (Howard-3, blue) uncertainties reduce SCC by 10% (at zero correlation), whereas for a linear damage function (Burke, green) uncertainties increase it by 13%. For the other η-ρ combination (Fig. 2b) the effects are +10% (DICE), +7% (Howard-3) and +14% (Burke). Also the effects of correlations are very different. For the quadratic damage function, correlations show no effect. But an increasing correlation implies for a linear damage function (green) a higher SCC, whereas for a third-order damage function (blue) a lower SCC. The dark stripes at the centers show that economic growth risk (*g*) increases the expected SSC, while emission risk (*E*) often decreases it.

In [Rennert *et al.*, 2022a+b], the SCC probability distributions have a standard deviation of 50% of the expectation value. Although this work concerns only two uncertainty distributions, results are similar. For the DICE case, the relative standard deviation is about 35%, whereas for Burke it is 40% and for Howard-3 60%.

[1] Assuming a global GDP of 100 T\$/y in 2025.

In reality there is a positive correlation between economic activity and greenhouse-gas emissions. We see that for the quadratic damage function, this correlation has no effect on the SCC, but for the steeper Howard-3 damage function, the correlation causes a reduction of the SCC. In other words, positive effects of higher economic growth cancel partially the negative effects of a warmer climate. If the world shifts to green economic growth, where technological innovations favor non-fossil energy sources, then the correlation might decrease or turn negative. In case of a steep damage function, this shift would drive up the SSC or, equivalently, enhance the benefits of greenhouse-gas mitigation.

In sum, this study stresses the importance of including correlated uncertainties in the calculation of the benefits of emissions reduction scenarios.

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