How to Calculate the Effects of Scenario Adjustments

Paul Alkemade

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ROECS, Rotterdam Office of Economic Climate Studies

and

Delft University of Technology

Rotterdam / Delft The Netherlands

p.f.a.alkemade@gmail.com

SSP scenarios

Emission table (GtCO$_2$/y) (AR6)

	SSP1 1.9	SSP1 2.6	SSP2 4.5	...	SSP3 7.0
2010	36.0	36.0	36.0	...	36.0
2020	39.0	39.0	39.0	...	45.0
2030	23.4	34.2	42.9	...	52.7
2040	10.2	26.2	~~43.6~~ 39.0	...	58.1
...	...	...	...	...	...
2100	-13.9	-9.8	~~9.1~~ 0.0	...	83.1

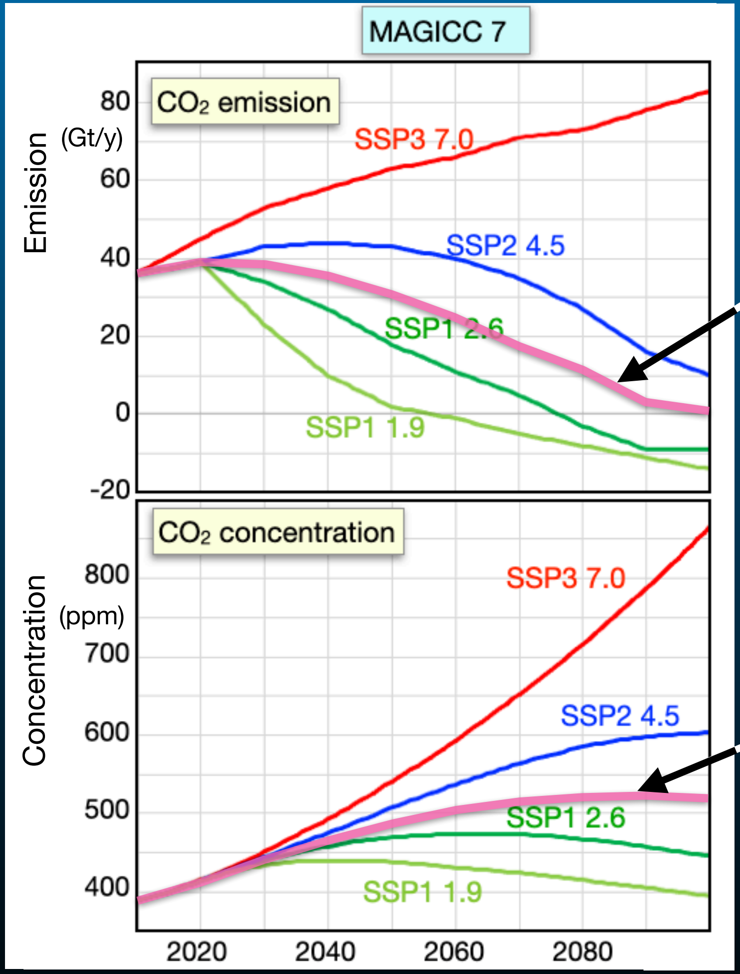
MAGICC →

CO$_2$ concentrations (ppm)

	SSP1 1.9	SSP1 2.6	SSP2 4.5	...	SSP3 7.0
2010	389	389	389	...	389
2020	413	413	413	...	415
2030	430	432	439	...	450
2040	433	450	?	...	489
...	...	...	...	...	...
2100	382	429	?	...	830

- How to change the numbers?
- How to calculate the CO$_2$ concentrations of modified scenarios?
- ESM / emulator / table interpolations?

Example: SSP1 1.9 , SSP1 2.6 , SSP2 4.5 & SSP3 7.0



[Meinshausen, 2020]

A new emission scenario...

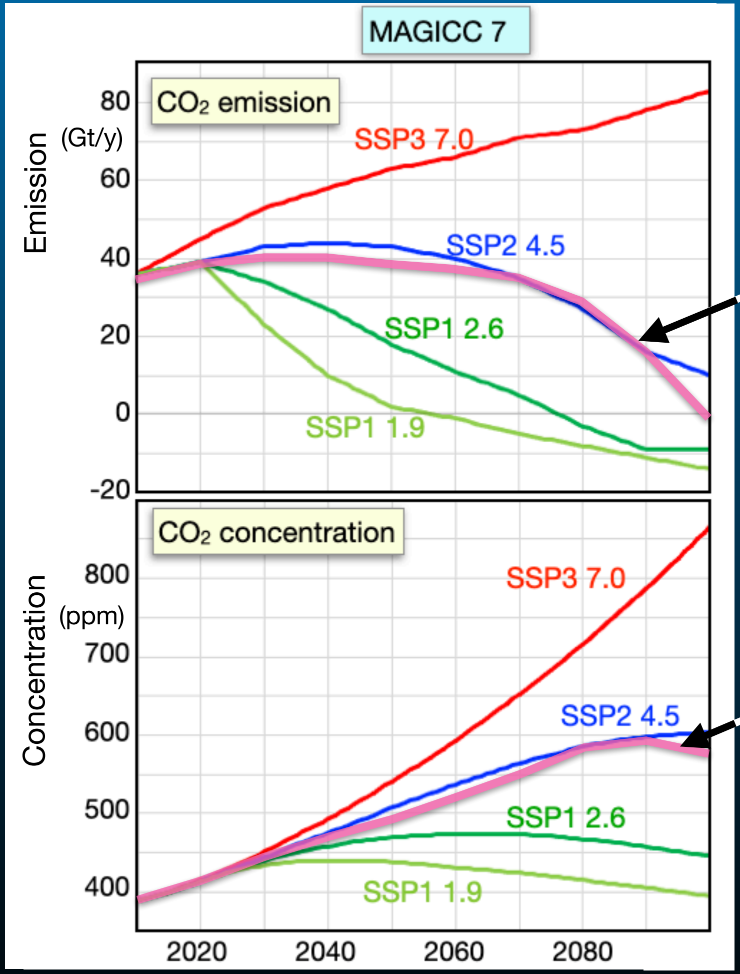
...exactly between SSP2 4.5 and SSP1 2.6

First guess for new concentration:

average of SSP2 4.5 and SSP1 2.6

No need for a new ESM
or MAGICC calculation

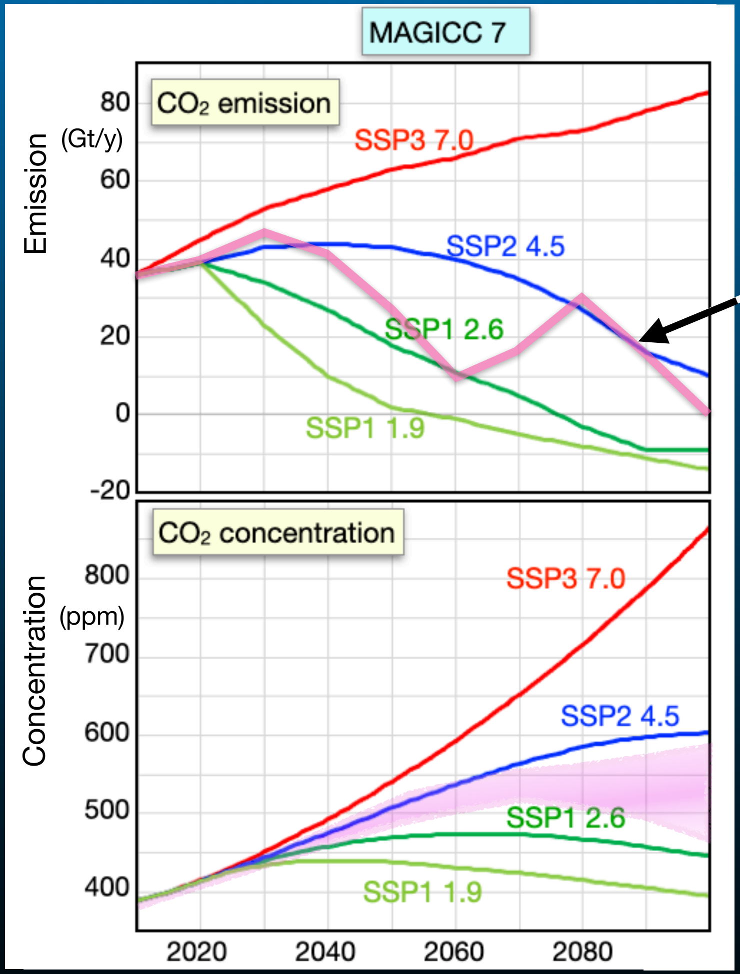
Example: SSP1 1.9 , SSP1 2.6 , SSP2 4.5 & SSP3 7.0



What about this emission scenario?

Like this?

Example: SSP1 1.9 , SSP1 2.6 , SSP2 4.5 & SSP3 7.0



And what about this emission scenario?

Good and easy interpolation is needed!

But how?

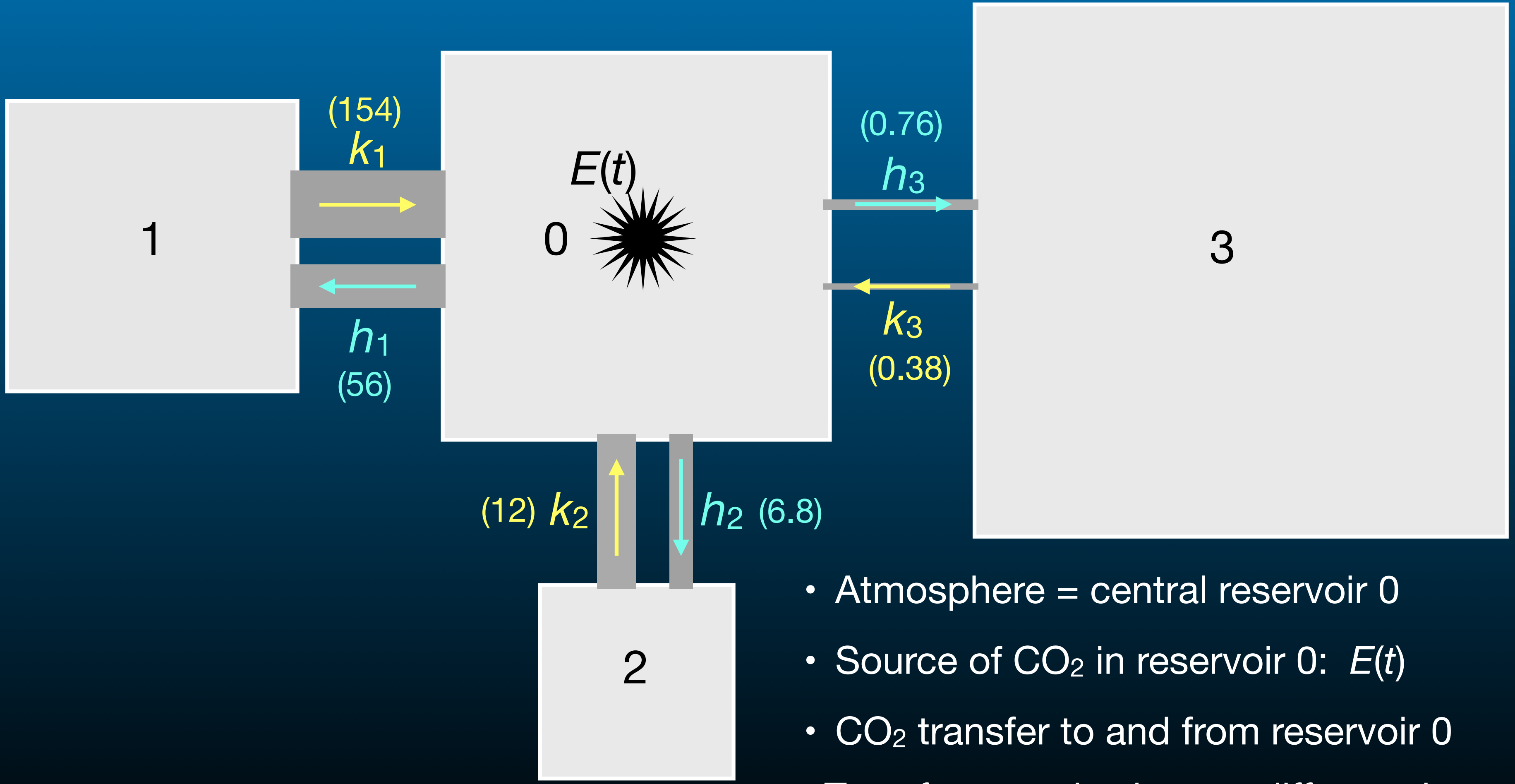
???

Subject of the presentation

Outline

- Introduction
- The Effective Reservoirs Method
- Calibrations (historic data & SSP scenarios)
- Replacement data tables by coefficient tables
- Application to new scenarios
- Discussion & Remarks
- Summary

Physical model for atmosphere with 3 *effective* reservoirs



- Atmosphere = central reservoir 0
- Source of CO_2 in reservoir 0: $E(t)$
- CO_2 transfer to and from reservoir 0
- Transfer rates h_j , k_j very different, but constant

(Note: h_j , k_j in 1000 x probability per year)

Mathematics: CO_2 in- and outflow of the reservoirs

Coupled set of 4 linear 1st-order differential equations:

CO_2 content $y^{(0)}(t)$ of reservoir 0 (atmosphere)

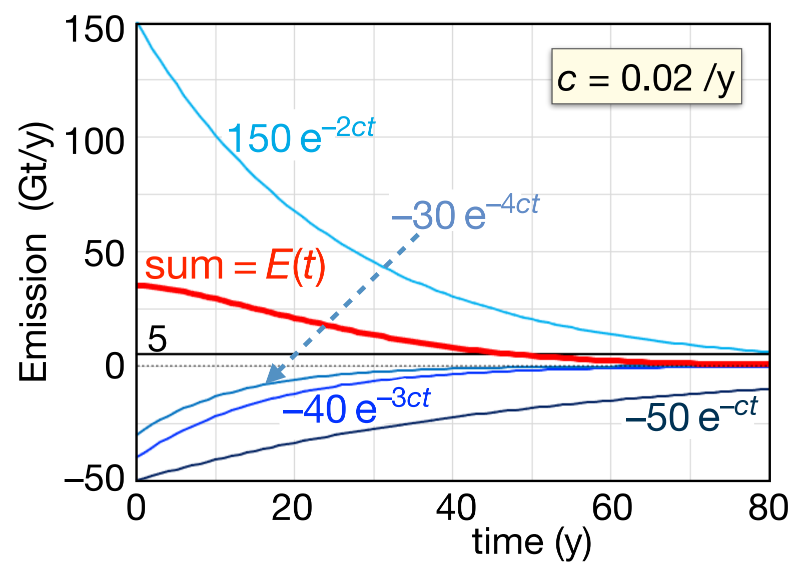
$$\frac{dy^{(0)}(t)}{dt} = \sum_{n=1}^{3} \underbrace{-h_n\, y^{(0)}(t)}_{\text{outflow}} + \sum_{n=1}^{3} \underbrace{k_n\, y^{(n)}(t)}_{\text{inflow}} + \underbrace{E(t)}_{\text{source}}$$

CO_2 content $y^{(n)}(t)$ of reservoirs 1, 2, and 3 $(=n)$

$$\frac{dy^{(n)}(t)}{dt} = \underbrace{+\, h_n\, y^{(0)}(t)}_{\text{inflow}} \underbrace{-\, k_n\, y^{(n)}(t)}_{\text{outflow}}$$

Mathematics: Laplace transform of emission function

Approximation for emission: $E(t) \cong \sum_{i=0}^{i_m} P_i \, e^{-ict}$



P_i (Gt/y):

$P_0 = 5$

$P_1 = -50$

$P_2 = 150$

$P_3 = -40$

$P_4 = -30$

Laplace transform of emission

Mathematics: CO_2 in- and outflow of the reservoirs

Solutions:

$$y^{(n)}(t) \cong \sum_{i=0}^{i_m} P_i\, f_i^{(n)} \exp(-ict) + \sum_{j=0}^{n_m} g_j^{(n)} \exp(-a_j t) + \sum_{i=0}^{i_m} \sum_{j=0}^{n_m} P_i\, H_{ij}^{(n)} \exp(-a_j t)$$

related to: emission
& starting values

CO_2 transfers

emission and transfers

Note the exponentials!

Effective Reservoirs (interpolation) Method

CO_2 content of the reservoirs:

$$y^{(n)}(t) \cong \sum_{i=0}^{i_m} P_i f_i^{(n)} \exp(-ict) + \sum_{j=0}^{n_m} g_j^{(n)} \exp(-a_j t) + \sum_{i=0}^{i_m} \sum_{j=0}^{n_m} P_i H_{ij}^{(n)} \exp(-a_j t)$$

New scenario = new set of prefactors P_i

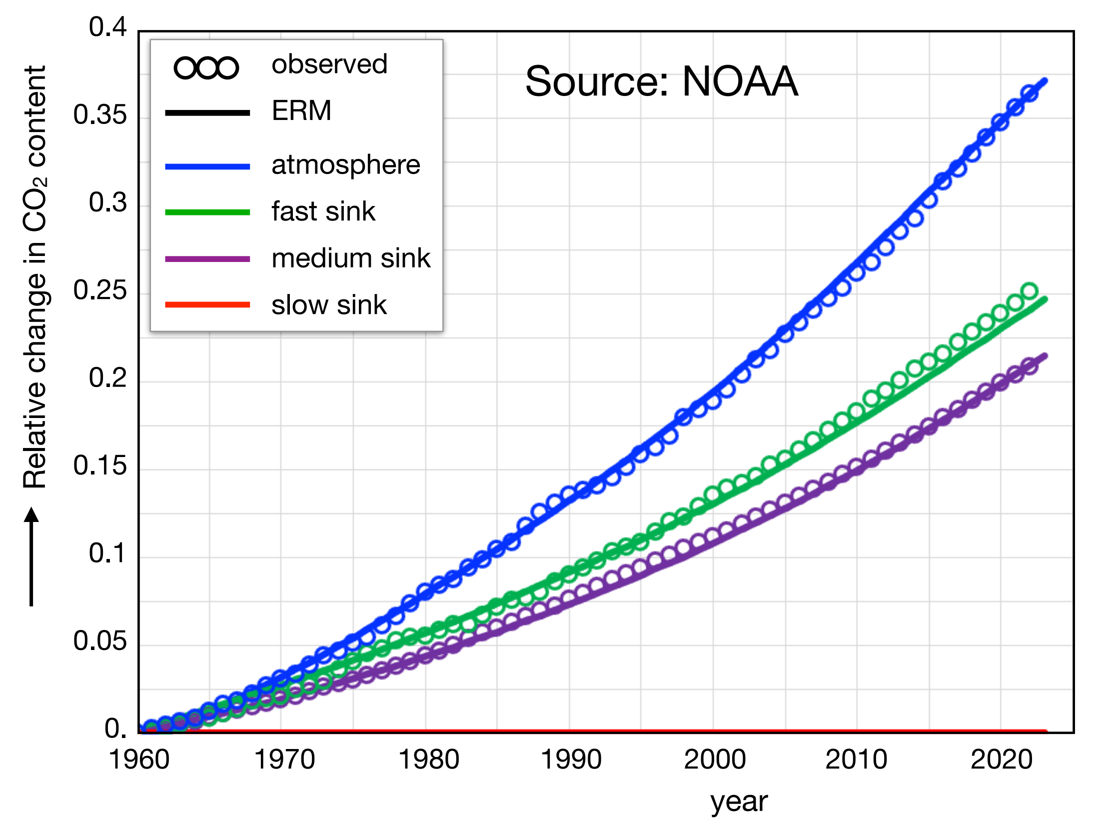
Solutions are linear functions!

Effective Reservoirs (interpolation) Method

Examples of applications:

- updating existing scenarios
- sensitivity analysis
- effects of policy modifications
- cost-benefit calculations
- effects of unforeseen events

Calibration of transfer rates from historic data (1960-2022)



Calibration results:

$h_1 = 0.091\ \mathrm{y}^{-1}$

$k_1 = 0.128\ \mathrm{y}^{-1}$

(fast)

$h_2 = 0.018\ \mathrm{y}^{-1}$

$k_2 = 0.014\ \mathrm{y}^{-1}$

(medium)

$h_3 = 0.000\ \mathrm{y}^{-1}$

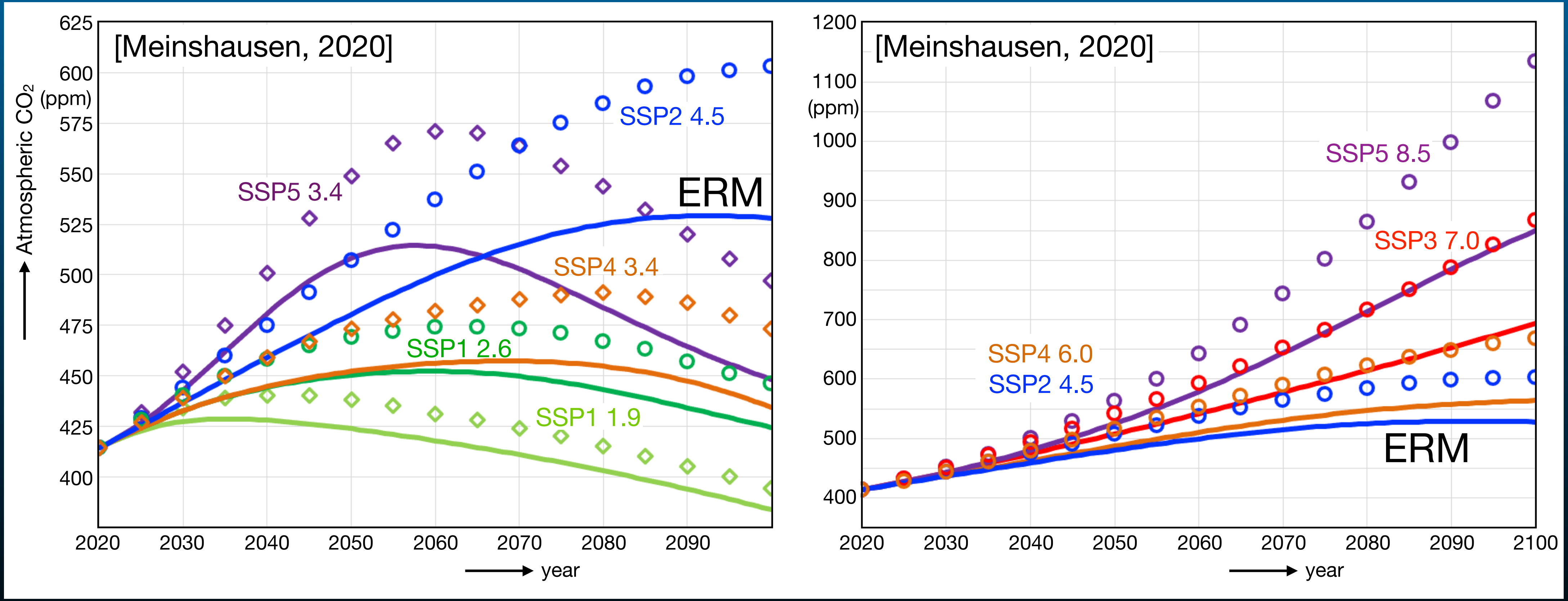
$k_3 = 0.000\ \mathrm{y}^{-1}$

(slow)

Application historic rates to SSP scenarios (2020-2100)

medium- and low-emission scenarios

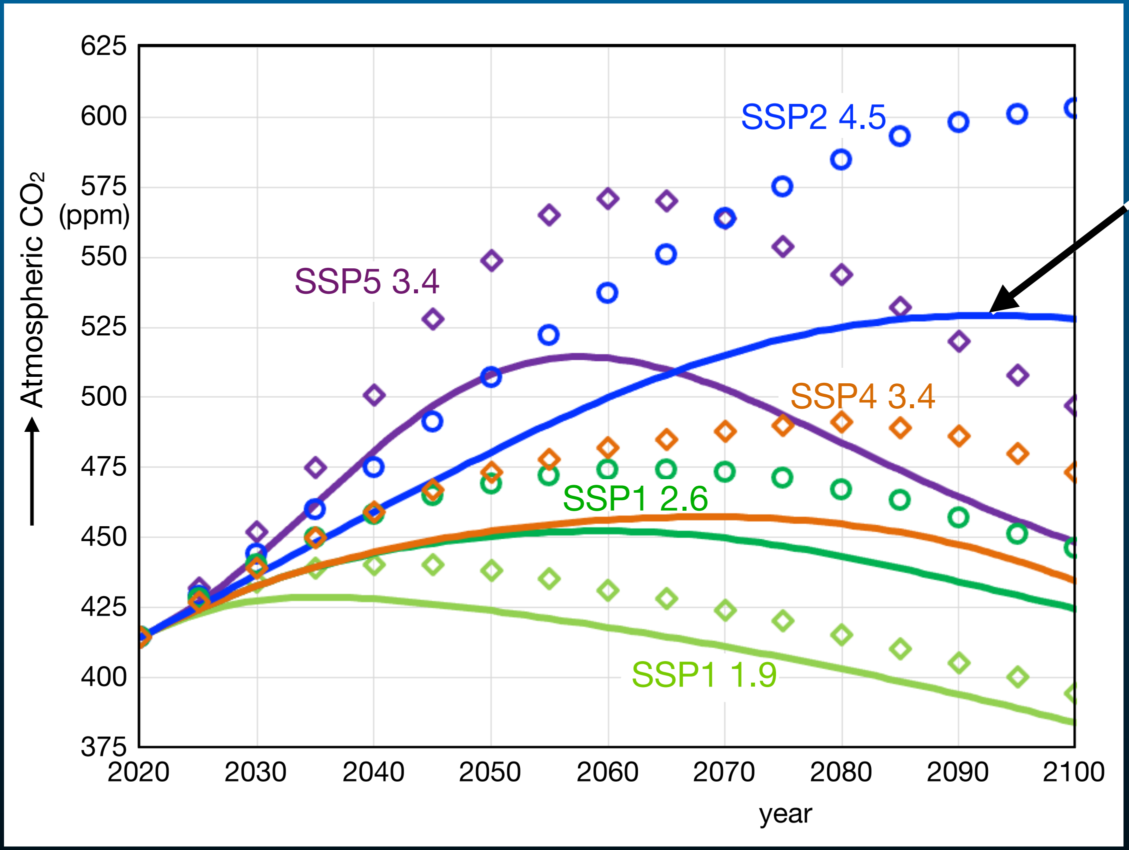
high-emission scenarios



No match! Historic calibration does not work

Calibration of transfer rates for medium and low SSP emissions

medium- and low-emission scenarios



with 1960-2022 calibration:

$h_1 = 0.091\ \mathrm{y}^{-1}$

$k_1 = 0.128\ \mathrm{y}^{-1}$

$h_2 = 0.018\ \mathrm{y}^{-1}$

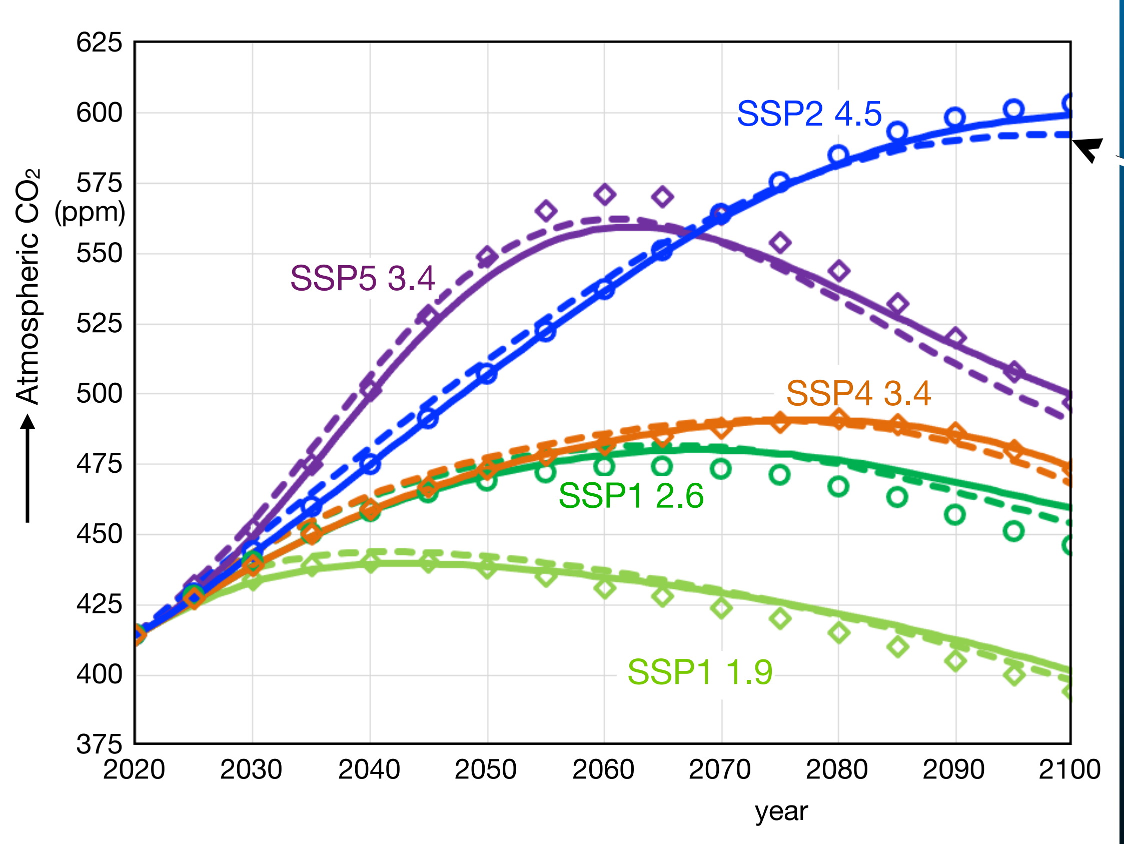
$k_2 = 0.014\ \mathrm{y}^{-1}$

$h_3 = 0.0\ \mathrm{y}^{-1}$

$k_3 = \ 0.0\ \mathrm{y}^{-1}$

Calibration of transfer rates for medium and low SSP emissions

medium- and low-emission scenarios



Reduced outflow from atmosphere

h_1 = ~~$0.091\ \mathrm{y}^{-1}$~~ $0.052\ \mathrm{y}^{-1}$

$k_1 = 0.128\ \mathrm{y}^{-1}$

h_2 = ~~$0.018\ \mathrm{y}^{-1}$~~ $0.010\ \mathrm{y}^{-1}$

$k_2 = 0.014\ \mathrm{y}^{-1}$

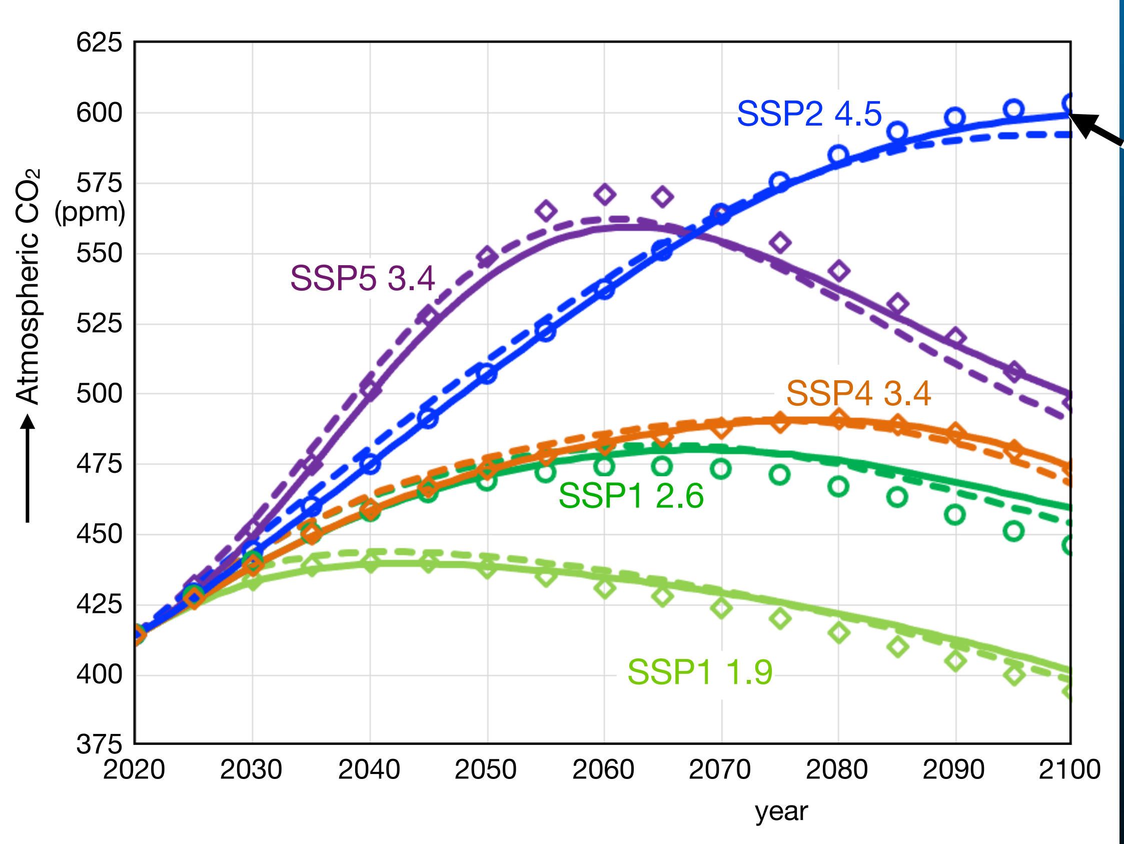
$h_3 = 0.0\ \mathrm{y}^{-1}$

$k_3 = 0.0\ \mathrm{y}^{-1}$

Note: one fit for 5 scenarios

Calibration of transfer rates for medium and low SSP emissions

medium- and low-emission scenarios



Best fit!

Increased back flow to atmosphere

$h_1 = 0.091\ \mathrm{y}^{-1}$

$k_1 =$ ~~$0.128\ \mathrm{y}^{-1}$~~ $0.405\ \mathrm{y}^{-1}$

$h_2 = 0.018\ \mathrm{y}^{-1}$

$k_2 =$ ~~$0.014\ \mathrm{y}^{-1}$~~ $0.031\ \mathrm{y}^{-1}$

$h_3 = 0.0\ \mathrm{y}^{-1}$

$k_3 = 0.0\ \mathrm{y}^{-1}$

Note: one fit for 5 scenarios

Matrix $H^{(0)}$

$$y^{(n)}(t) \cong \sum_{i=0}^{i_m} P_i\, f_i^{(n)} \exp(-ict) + \sum_{j=0}^{n_m} g_j^{(n)} \exp(-a_j t) + \sum_{i=0}^{i_m} \sum_{j=0}^{n_m} P_i \boxed{H_{ij}^{(n)}} \exp(-a_j t)$$

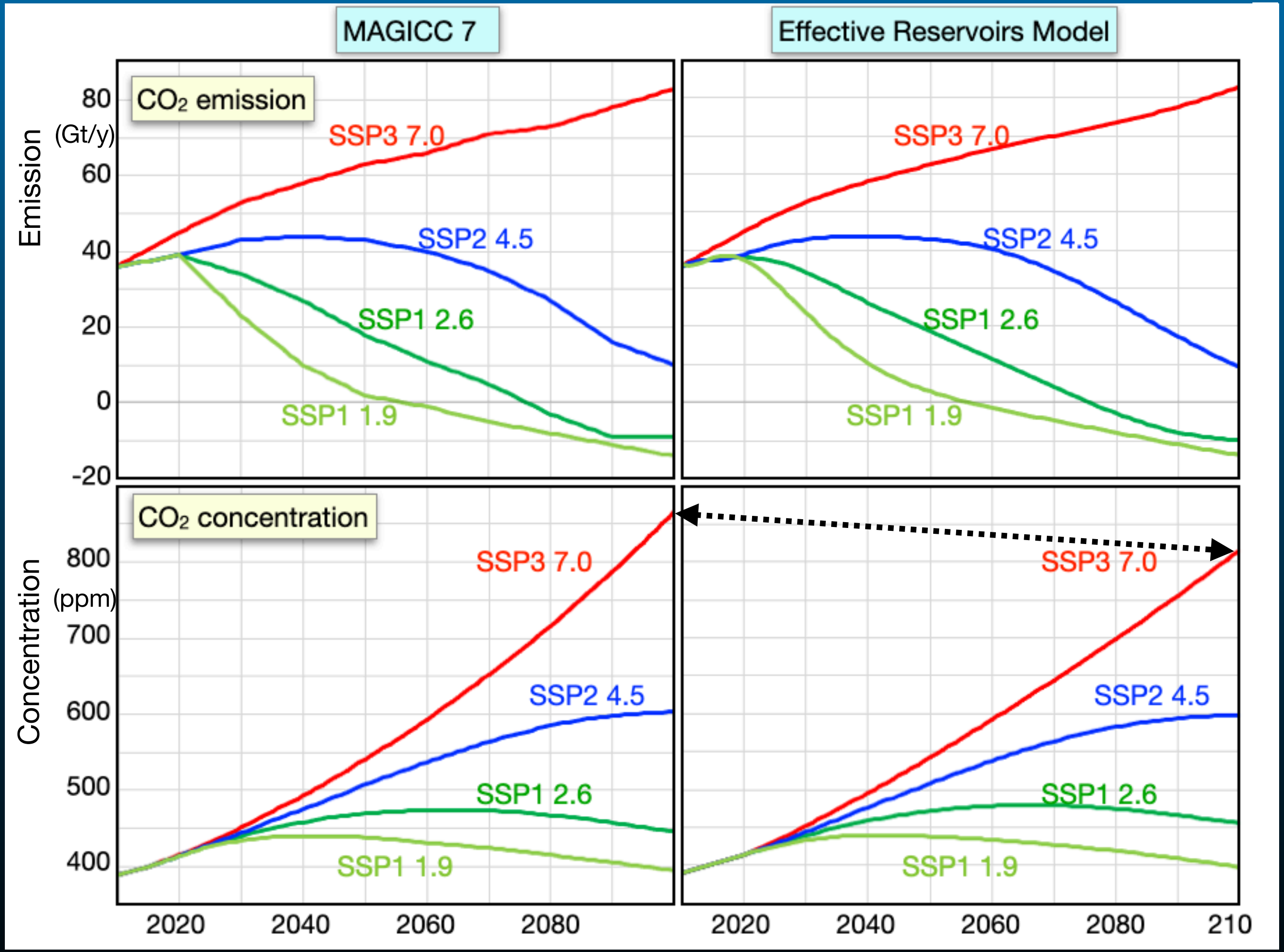
$y_{\mathrm{SSP}}(t)$ in ppm :

	SSP1 1.9	SSP1 2.6	SSP2 4.5	...	SSP3 7.0
2010	389	389	389	...	389
2020	413	413	413	...	415
2030	430	432	439	...	450
2040	433	450	465	...	489
...	...	...	...	...	...
2100	382	429	561	...	830

	$j = 0$	1	2	3
$i = 0$	26365.134	–0.391	–5.407	32888.565
1	13.183	–0.408	–9.617	14.706
2	6.591	–0.426	–43.503	7.353
3	4.394	–0.445	17.238	4.902
4	3.296	–0.466	7.194	3.676
5	2.637	–0.489	4.545	2.941
6	2.197	–0.515	3.322	2.451
7	1.883	–0.544	2.618	2.101
8	1.648	–0.576	2.160	1.838

- For given t_0 (2020), starting conditions, and calibration: $H^{(0)}$ matrix elements for all scenarios are the same
- General $H^{(0)}$ matrix replaces specific SSP tables

Application to new scenarios



Laplace transform of emissions

Solved differential equations

Application to new scenarios

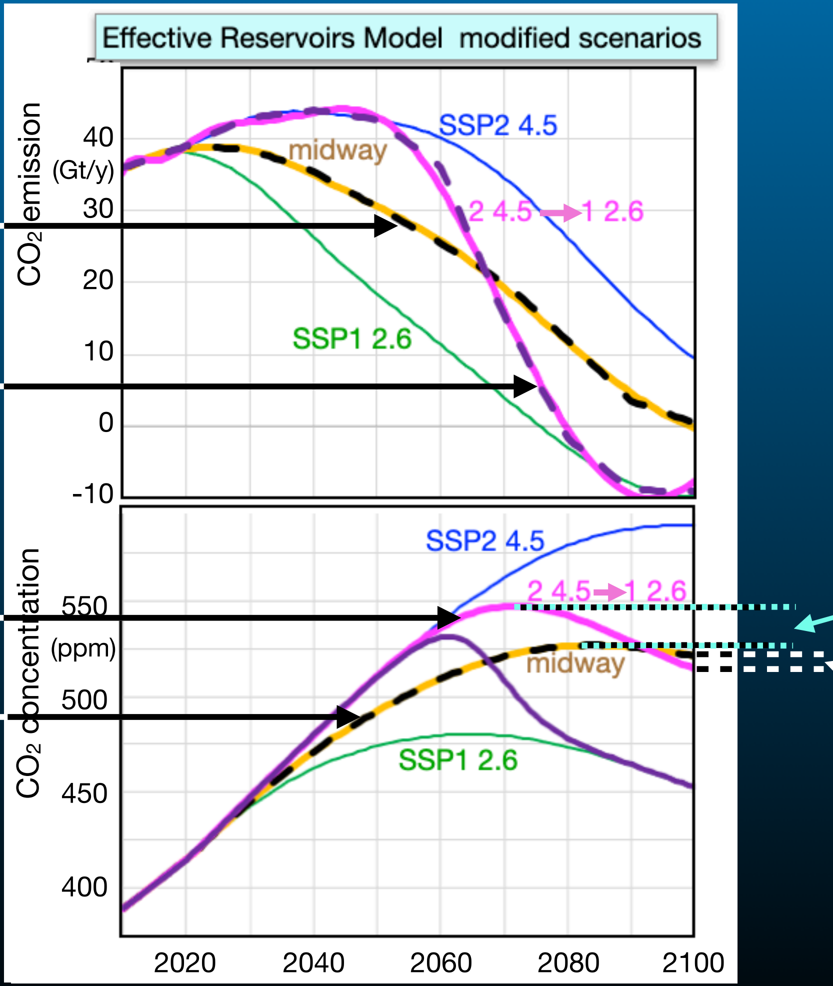
Midway between SSP2 4.5 and SSP1 2.6

Halfway transition from SSP2 4.5 to SSP1 2.6

(same cumulative emission in 2100)

Halfway transition from SSP2 4.5 to SSP1 2.6

Midway between SSP2 4.5 and SSP1 2.6



+0.17 K

–0.07 K

2100-CO_2 concentration: 8 ppm lower

max CO_2 concentrations: 20 ppm higher

Discussion & Remarks

Effective Reservoirs Method:

- Transfer rates depend on mean temperature in scenario
- Dynamic transfer rates possible via recursive calculations (?)
- Can be used for other GHGs (CH$_4$, NO$_2$, ...)
- Extension to forcings and global temperatures
- Physical interpretation of calibrated rates remains open
- No need for more reservoirs / nor transfers between reservoirs
- Open source, *e.g.* via excel or python

Summary

Effective Reservoirs Method (ERM) is a clever interpolation
to extend the set of scenarios from *Earth System Modelling* (emulators)

It is based on physical laws of connected *effective* CO_2 reservoirs...
...and the mathematical Laplace transforms

ERM is described by a fixed matrix H_{ij} plus vectors f_i, g_j, a_j

Useful for interpolation of low- and medium-emission scenarios.

Thank you